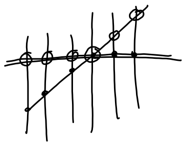
3.5 Discontinuities

1. Show that $f(x) = \begin{cases} x & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}$

is not Riemann integrable.



$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^n f(c_i)\Delta x$

$U_p = \sum_{i=1}^n f(M_i)\Delta x$; $L_p = \sum_{i=1}^n f(m_i)\Delta x$

if $\inf(U_p) = \sup(L_p) = \int$

Let P be a partition. Each interval contains both a rational & irrational #, so

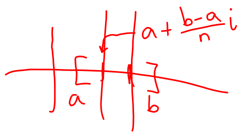
for $\begin{cases} x < 0, & m_i = x_i \text{ \& } M_i = 0 \\ x \geq 0, & m_i = 0,\ M_i = x_i \geq 0 \end{cases}$

$$U_p = \sum_{i=1}^n f(M_i)\Delta x = \sum_{i=1}^n 0\cdot\Delta x + \sum_{i=1}^n x_i\cdot\Delta x > C$$

$$L_p = \sum_{i=1}^n f(m_i)\Delta x = \sum_{i=1}^n x_i\Delta x + \sum_{i=1}^n 0\,\Delta x < 0$$

$U_p = \sum_{i=1}^n (a+\Delta x\cdot i)\cdot\Delta x$

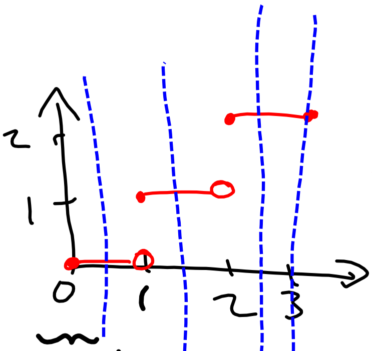
$L_p = \sum_{i=1}^n$



U_p has no inf (∞)

L_p has no sup $(-\infty)$

Prove that $f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \\ 2, & 2 \leq x \leq 3 \end{cases}$ is R-int.



$$\inf \sum_{i=1}^n f(M_i)\Delta x \stackrel{?}{=} \sup \sum_{i=1}^n f(m_i)\Delta x$$

$$3 = 3$$

$\inf\{3, (4,6)\}$ $\sup\{(0,2), 3\}$

0.5 $x0 + 1\cdot1 + 1.2 + 0.5\cdot2$

$(0, 1.5)$ $0 + 1.5$ $(1.5, 3)$