Ch 9, #13 : Prove by induction on n.

$$(n+1)\sum_{i=1}^{n} i^k = \sum_{i=1}^{n} i^{k+1} + \sum_{p=1}^{n}\left(\sum_{i=1}^{p} i^k\right)$$

I. "initial condition"

$n=1$

$(2)1 = 1 + 1 \Rightarrow 2 = 2$ ✓

II. "induction step"

assume it works for $n=j$

$$(j+1)\sum_{i=1}^{j} i^k = \sum_{i=1}^{j} i^{k+1} + \sum_{p=1}^{j}\left(\sum_{i=1}^{p} i^k\right)$$

to show it works for $n=j+1$

$$(j+1+1)\sum_{i=1}^{j+1} i^k \overset{?}{=} \sum_{i=1}^{j+1} i^{k+1} + \sum_{p=1}^{j+1}\left(\sum_{i=1}^{p} i^k\right)$$

$$(j+1+1)\sum_{i=1}^{j+1} i^k = (j+1)\left[\sum_{i=1}^{j} i^k + (j+1)^k\right] + \left[\sum_{i=1}^{j} i^k + (j+1)^k\right]$$

$$\sum_{i=1}^{j+1} i^k = \sum_{i=1}^{j} i^k + (j+1)^k$$

$$= (j+1)\left(\sum_{i=1}^{j} i^k\right) + (j+1)^{k+1} + \sum_{i=1}^{j} i^k + (j+1)^k$$

$$= \sum_{i=1}^{j} i^{k+1} + \sum_{p=1}^{j}\left(\sum_{i=1}^{p} i^k\right) + (j+1)^{k+1} + \sum_{i=1}^{j} i^k + (j+1)^k$$

$$= \sum_{i=1}^{j+1} i^{k+1} + \sum_{p=1}^{j}\left(\sum_{i=1}^{p} i^k\right) + \underbrace{\sum_{i=1}^{j} i^k + (j+1)^k}$$

$$= \sum_{i=1}^{j+1} i^{k+1} + \underbrace{\sum_{p=1}^{j}\left(\sum_{i=1}^{p} i^k\right) + \sum_{i=1}^{j+1} i^k}$$

$$= \sum_{i=1}^{j+1} i^{k+1} + \sum_{p=1}^{j+1}\left(\sum_{i=1}^{p} i^k\right)$$ ☑ groovy

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\hookrightarrow 1^2+2^2+3^2+4^2+\ldots+n^2$$

$n=1$

$$\sum_{i=1}^{1} i^2 = \frac{1(1+1)(2+1)}{6}$$

$$1 = \frac{6}{6} \quad \checkmark$$

assume

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

to show

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\sum_{i=1}^{k+1} i^2 = \underbrace{1^2+2^2+3^2+\ldots+k^2}+(k+1)^2$$

$$= \sum_{i=1}^{k} i^2 + (k+1)^2$$

$$= \frac{k(k+1)(2k+1)}{6} + \frac{(k+1)^2 \cdot 6}{6}$$

$$= \frac{(k+1)\left[\overset{2k^2+7k+6}{2k^2+k+6k+6}\right]}{6}$$

$$= \frac{(k+1)(k+2)(2k+3)}{6}$$

$$= \frac{(k+1)(k+1+1)(2(k+1)+1)}{6} \quad \checkmark$$