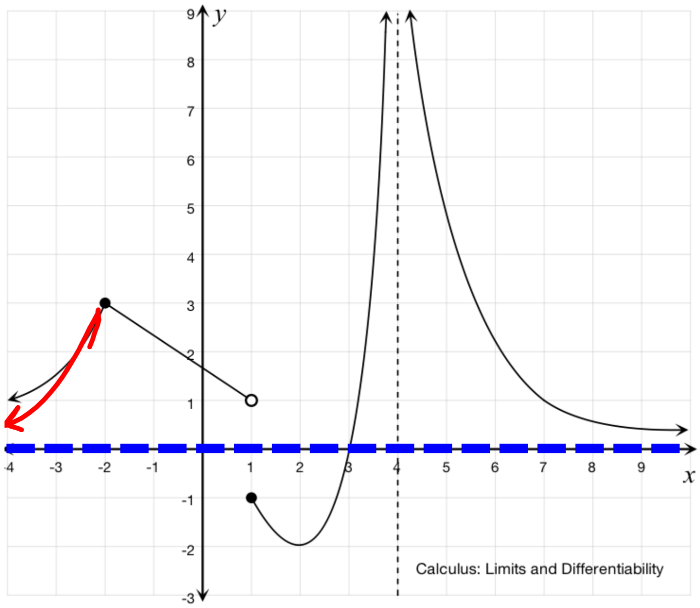
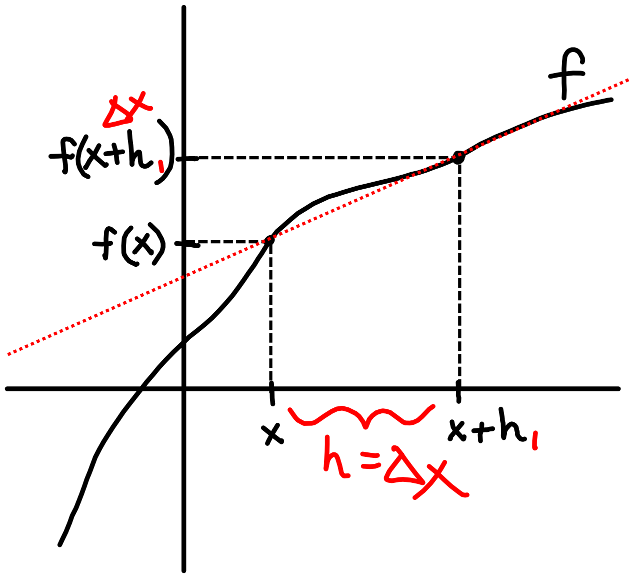


as $x \to c$ approaches..	$f(x)$ approaches..
-2	3
1^- (from the left)	1
1^+ (from the right)	-1
3	0
$-\infty$	0
∞	0
4	∞ (does not exist)

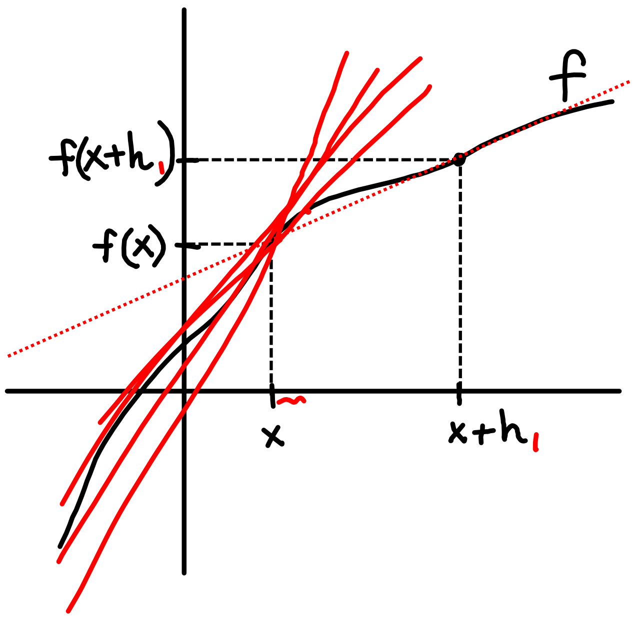
$\lim_{x \to c} f(x)$



secant line

slope :

$$\frac{f(x+\Delta x)-f(x)}{x+\Delta x - x} = \frac{f(x+\Delta x)-f(x)}{\Delta x}$$



tangent line

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

@ $x = c$

$$\lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

1.2

$$f(x) = \frac{x-2}{x^2-4}, \quad x \neq 2, -2$$

$$\lim_{x \to 2} f(x) = \lim \frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)} = \frac{1}{4}$$

What happens to *f(x)* as *x* approaches 2?

x	1.9	1.99	1.999	2	2.001	2.01	2.1
f(x)							

Informal Description of the Limit

If *f(x)* becomes arbitrarily close to a single number *L* as *x* approaches *c* from either side, the limit of *f(x)*, as *x* approaches *c*, is *L*.

$$\lim_{x \to c} f(x) = L$$

Note: the existence or nonexistence of *f(x)* at *x=c* has no bearing on the existence of the limit as *x* approaches *c*.

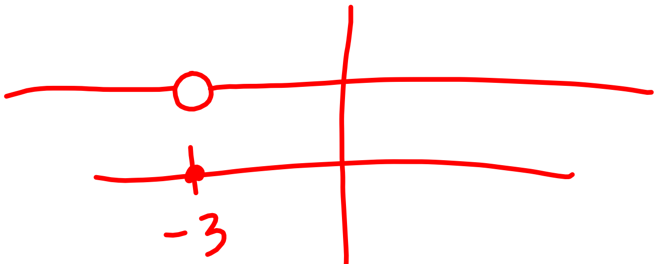
A function can be undefined for a certain value of *c* with the limit as *x* approaches *c* still defined.

$$\lim_{x \to -3} \frac{\sqrt{1-x} - 2}{x+3} = -0.25$$

$$-x-3 = -1(x+\cancel{3})$$

$$\frac{\sqrt{1-x} - 2}{x+3} \cdot \frac{\sqrt{1-x} + 2}{\sqrt{1-x} + 2} = \frac{1-x-4}{(\cancel{x+3})(\sqrt{1-x}+2)}$$

$$f(x) = \begin{cases} 1, & x \neq -3 \\ 0, & x = -3 \end{cases}$$



$$\lim_{x \to -3} f(x) = 1$$

$$\lim_{x \to 0} \frac{|2x|}{x}$$

$$|f(x)| = \begin{cases} f(x), & f(x) \geq 0 \\ -f(x), & f(x) < 0 \end{cases}$$

$$\frac{|2x|}{x} = \begin{cases} \frac{2x}{x} = 2, & 2x > 0 \\ & x > 0 \\ \frac{-2x}{x} = -2, & 2x < 0 \\ & x < 0 \end{cases}$$

$$\lim_{x \to 0^+} f(x) = 2$$

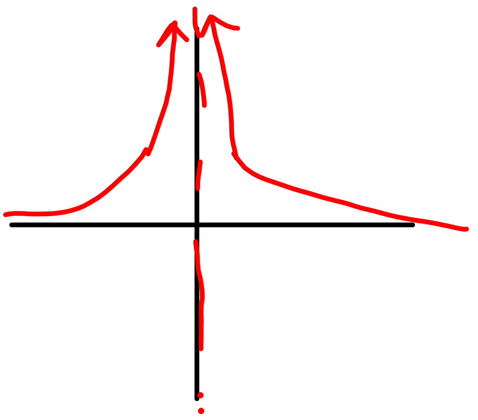
$$\lim_{x \to 0^-} f(x) = -2$$

$\lim_{x \to 0} f(x) =$ does not exist

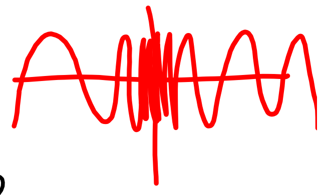
$$\lim_{x \to 3^+} \frac{-2|3-x|}{x-3} = 2$$

$$\frac{-2|3-x|}{x-3} = \begin{cases} \frac{-2(3-x)}{x-3} = 2, & 3-x \geq 0 \\ \frac{-2(-(3-x))}{x-3} = -2 & 3-x < 0 \end{cases}$$

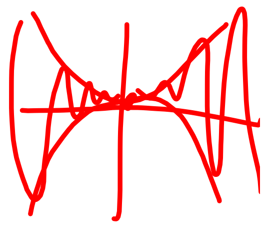
$$\lim_{x \to 0} \frac{1}{x^4} = \infty$$



$\lim_{x \to 0} \sin \frac{1}{x}$ does not exist



x	$\frac{2}{\pi}$	$\frac{2}{3\pi}$	$\frac{2}{5\pi}$	$\frac{2}{7\pi}$	$\frac{2}{9\pi}$	$\frac{2}{11\pi}$
$\sin \frac{1}{x}$	1	-1	1	-1	1	-1

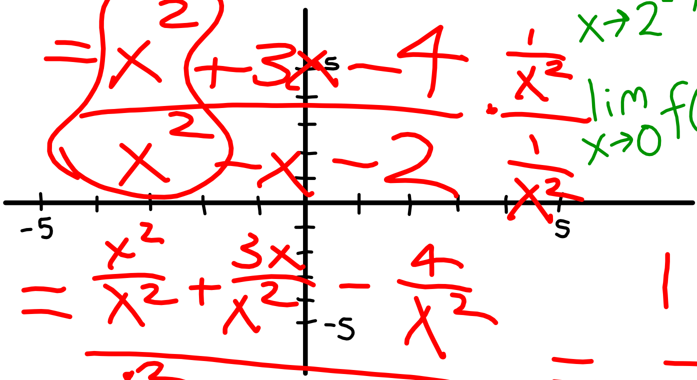


"Dirichlet Function"

$$f(x) = \begin{cases} 0, & \text{if } x \text{ is rational} \\ 1, & \text{if } x \text{ is irrational} \end{cases}$$

Graph the rational function.

$$f(x) = \frac{(x+4)(x-1)}{(x-2)(x+1)}$$



$$= \frac{x^2+3x-4}{x^2-x-2} \cdot \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

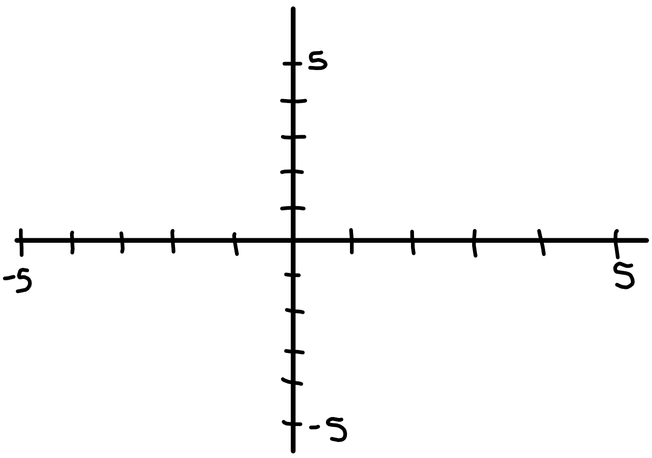
$$= \frac{\frac{x^2}{x^2}+\frac{3x}{x^2}-\frac{4}{x^2}}{\frac{x^2}{x^2}-\frac{x}{x^2}-\frac{2}{x^2}} = \frac{1+\underbrace{\frac{3}{x}}_{\to 0}-\underbrace{\frac{4}{x^2}}_{\to 0}}{1-\underbrace{\frac{1}{x}}_{\to 0}-\underbrace{\frac{2}{x^2}}_{\to 0}} = 1$$

$$\lim_{x\to\infty} f(x) = 1$$

$$\lim_{x\to 2^-} f(x) = -\infty$$

$$\lim_{x\to 0} f(x) = 2$$

$$f(x) = \frac{x(x-2)}{x+3}$$



$$\lim_{x\to -\infty} f(x) = \lim_{x\to -\infty} \frac{x^2}{x} = -\infty$$

$$\lim_{x\to -3^+} f(x) = \infty$$

$$\lim_{x\to 2} f(x) = 0$$

Building up to the $\epsilon - \delta$ Definition of the Limit

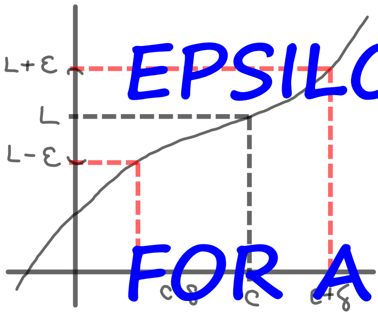
ε = epsilon

δ = delta

Translating the "informal description": $\lim_{x\to c} f(x) = L$

If $f(x)$ becomes arbitrarily close to a single number L as x approaches c from either side, the limit of $f(x)$, as x approaches c, is L.

DO NOT NEED TO KNOW EPSILON-DELTA FOR AP EXAM!



"$f(x)$ becomes arbitrarily close to L"

$f(x)$ lies in the interval $(L-\varepsilon, L+\varepsilon)$ for some (really small) $\varepsilon > 0$.

$|f(x) - L| < \varepsilon$

"the distance between $f(x)$ and L is less than ε"

"x approaches c"

There exists a (very small) positive number δ such that x is either in the interval $(c-\delta, c)$ or $(c, c+\delta)$.

$0 < |x - c| < \delta$

The first inequality guarantees that $x \neq c$.

1.3 Evaluating Limits Analytically

If $\lim_{x\to c} f(x) = f(c)$,

we say that $f(x)$ is continuous at c.

Evaluating Limits Analytically

Basic Limits

Let $b, c \in \mathbb{R}$, $n > 0$ *an integer*, $f, g -$ *functions*, $\lim_{x\to c} f(x) = L$, $\lim_{x\to c} g(x) = K$

1. Constant $\lim_{x\to c} b = b$
2. Identity $\lim_{x\to c} x = c$
3. Polynomial $\lim_{x\to c} x^n = c^n$
4. Scalar Multiple $\lim_{x\to c}[bf(x)] = bL$
5. Sum or Difference $\lim_{x\to c}[f(x) \pm g(x)] = L \pm K$
6. Product $\lim_{x\to c}[f(x)g(x)] = LK$
7. Quotient $\lim_{x\to c}\left[\frac{f(x)}{g(x)}\right] = \frac{L}{K}$, $K \neq 0$
8. Power $\lim_{x\to c}[f(x)]^n = L^n$

$\lim\limits_{x\to c}[f(x) \pm g(x)] = \lim\limits_{x\to c} f(x) \pm \lim\limits_{x\to c} g(x)$

$\lim\limits_{x\to c}[f(x)g(x)] = \left[\lim\limits_{x\to c} f(x)\right] \cdot \left[\lim\limits_{x\to c} g(x)\right]$

$\lim\limits_{x\to c} \frac{f(x)}{g(x)} = \frac{\lim\limits_{x\to c} f(x)}{\lim\limits_{x\to c} g(x)}$, $\lim\limits_{x\to c} g(x) \neq 0$

$\lim\limits_{x\to c}[f(x)]^n = \left[\lim\limits_{x\to c} f(x)\right]^n$

Note: If substitution yields $\frac{0}{0}$, an indeterminate form, the expression must be rewritten in order to evaluate the limit.

If $\lim\limits_{x\to c} \frac{f(x)}{g(x)}$ yields

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $\frac{-\infty}{\infty}$, $\frac{\infty}{-\infty}$, or $\frac{-\infty}{-\infty}$

then

$$\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$$

$$\lim_{x \to c} a = a$$

$$\lim_{x \to c} x = c$$

$$\lim_{x \to c} x^n = c^n$$

$$\lim_{x \to 5} (-3) = -3$$

$$\lim_{x \to -\pi} x = -\pi$$

$$\lim_{x \to -1} x^5 = -1$$

1.3

12. $\lim_{x \to 1} \left(3x^3 - 2x^2 + 4\right)$

18. $\lim_{x \to 3} \dfrac{\sqrt{x+1}}{x-4}$

30. $\lim_{x\to 1} \sin \frac{\pi x}{2} =$

36. $\lim_{x\to 7} \sec\left(\frac{\pi x}{6}\right)$

38. $\lim_{x\to c} f(x) = \frac{3}{2}$; $\lim_{x\to c} g(x) = \frac{1}{2}$

(a) $\lim_{x\to c} \left[4f(x)\right] = 4\left(3/2\right)$

(b) $\lim_{x\to c} \left[f(x)+g(x)\right] = \frac{3}{2} + \frac{1}{2}$

(c) $\lim_{x\to c} \left[f(x)g(x)\right] = \left(\frac{3}{2}\right)\left(\frac{1}{2}\right)$

(d) $\lim_{x\to c} \frac{f(x)}{g(x)} = \frac{3/2}{1/2}$