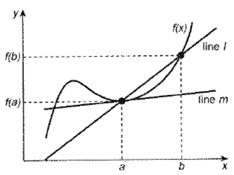
1. B	6. A	11. D
2. E	7. A	12. D
3. C	8. D	13. C
4. B	9. E	14. E
5. B	10. B	15. A

REVIEW QUESTIONS

1. Below is the graph of the function $y = f(x)$, line l passing through the points $(a, f(a))$ and $(b, f(b))$ on the graph of f, and line m which is tangent to the graph of f at $(a, f(a))$.



Which of the following statements is true?

True if "the slope of line m"

I. Line m is the derivative of f at $x = a$. F

II. The slope of line l is the average rate of change of f from a to b. T

III. The slope of line m is given by $\lim_{\Delta x \to 0} \frac{f(a + \Delta x) - f(a)}{\Delta x}$. T

(A) I only
(B) III only
(C) I and III
(D) II and III
(E) All of the above

2. Suppose $f(x)$ is not differentiable at $x = a$. Which of the following must be true?

(A) $f(x)$ is not defined at $x = a$ but could be continuous at $x = a$. F

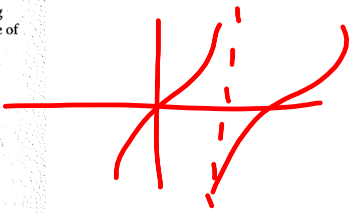
(B) ~~$f(x)$ is not continuous at $x = a$ but could be defined at $x = a$.~~

(C) ~~$f(x)$ is defined at $x = a$ but is not continuous at $x = a$.~~

(D) ~~$f(x)$ is continuous and defined at $x = a$.~~

(E) $f(x)$ could be defined and continuous at $x = a$.

could be T
T/F
T/F
T/F



3. Suppose the derivative of the function $f(x)$ exists at $x = a$. Which of the following expressions is NOT equal to the derivative of f at a?

(A) $\lim_{h \to 0} \frac{f(a + h) - f(a)}{h}$ ✓

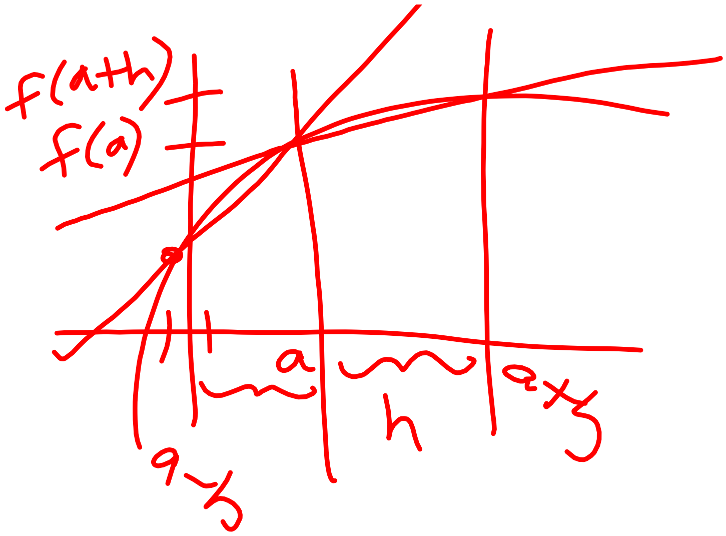
(B) $\lim_{h \to 0} \frac{f(a) - f(a - h)}{h}$

(C) $\lim_{h \to 0} \frac{f(a - h) - f(a)}{h}$

(D) $\lim_{h \to 0} \frac{f(a) - f(a + h)}{-h}$ ✓

(E) $\lim_{b \to a} \frac{f(a) - f(b)}{a - b}$

$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$



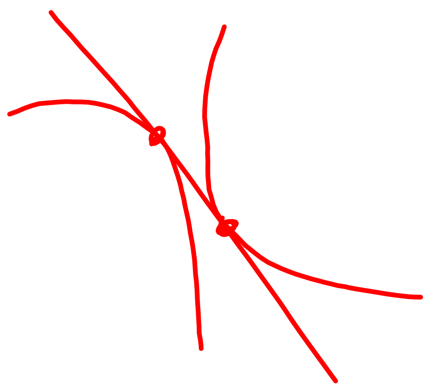
$$\frac{f(a)-f(a-h)}{\underbrace{a-(a-h)}_{=h}}$$

$$\frac{f(a-h)-f(a)}{\underbrace{a-h-a}_{=-h}}$$

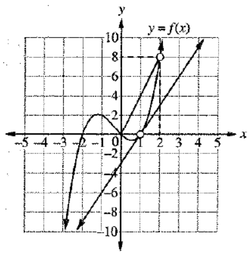
4. If $f'(6) = -2$, which of the following must be true?

I. f is decreasing at $x = 6$ ✓
II. f is continuous at $x = 6$ ✓
III. f is concave up at $x = 6$

(A) I and II only
(B) I and III only
(C) II and III only
(D) I, II and III
(E) None of the above



5. Which of the following statements is true for the function $y = f(x)$ whose graph is shown below?



slope of secant line is $\frac{f(2)-f(0)}{2-0}$

slope of tangent line @ $x=1$ is $f'(1)$

(A) The instantaneous rate of change of f at $x = 1$ is greater than the average rate of change of f on the interval $[0,2]$. F

(B) $\boxed{f'(1) < \frac{f(2)-f(0)}{2-0}}$ T

(C) $f'(1) = 0$ F

(D) The average rate of change of f on $[0,2]$ is 8. F

(E) None of the above

6. Which of the following statements must be true for the differentiable function $f(x)$?

I. The average rate of change of $f(x)$ is given by $f'(x)$ F

II. $\lim_{x \to a} f(x) = f(a) \forall a \in \mathbb{R}$ T def of continuity diff ⇒ cont.

III. $\lim_{x \to a} \frac{f(x)-f(a)}{x-a}$ exists for all real values of a. T def. of differentiability

(A) I and II only

(B) I and III only

(C) $\boxed{\text{II and III only}}$

(D) I, II and III

(E) None of the above

7. Suppose that the position of a particle is given by the differentiable function $x(t)$. Which of the following statements must be true?

   I. $\lim\limits_{h\to 0}\frac{x(t+h)-x(t)}{h}$ gives the velocity of the particle at any time t. T

   II. $\lim\limits_{t\to 3}\frac{x(t)-x(3)}{t-3}$ gives the velocity of the particle at time t = 3. T

   III. $\frac{x(4)+x(1)}{2}$ gives the average velocity on the interval [1,4] F

   (A) I and II only
   (B) I and III only
   (C) II and III only
   (D) I, II and III
   (E) None of the above

$x'(t)=v(t)$

$x'(3)=v(3)$

$V_{avg}=\frac{x(4)-x(1)}{4-1}$

8. Which of the following statements about the derivative of f at x_0 is false?

   (A) The derivative of f at x_0 is the instantaneous rate of change of f at x_0. T
   (B) The derivative of f at x_0 is the limit of a difference quotient. T
   (C) The derivative of f at x_0 is the slope of the tangent to f at x_0. T
   (D) The derivative of f at x_0 is the limit of the average rate of change of f on an interval as the length of the interval approaches zero. T
   (E) All of the previous statements are true.

$\lim\limits_{a\to b}\frac{f(a)-f(b)}{a-b}$

9. Which of the following expressions would NOT give the slope of the differentiable function f at a?

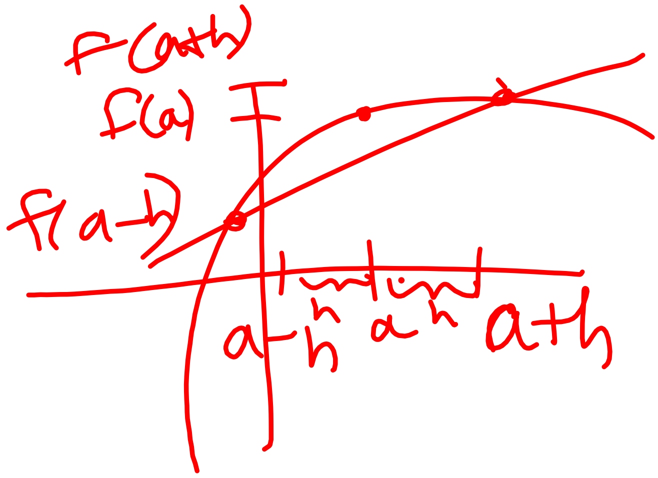
(A) $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ ✓

(B) $\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$ ✓

(C) $\lim_{x\to a}\frac{f(x)-f(a)}{h}$

(D) $\lim_{h\to 0}\frac{f(a+h)-f(a-h)}{2h}$

(E) $\lim_{x\to a}\frac{f(a)-f(x)}{a-x}$ ✓



10. Consider the function $c(t)$ representing the costs incurred by a painting company since the company's first year of business, where c is measured in $\frac{\$}{m^2}$ and t is measured in years since the company was founded. $c'(4)$ represents:

(A) ~~The cost, in $\frac{\$}{m^2}$ that the company had four years after it was founded.~~

(B) ~~The total cost, in \$, that the company incurred during it's fourth year of business.~~

(C) ~~The total cost, in \$, that the company incurred in it's first four years of business.~~

(D) ~~The average rate of change in costs in the company's first four years.~~

(E) The rate at which the company's costs were changing after four years of business.

$c(t) = \$/m^2$ cost

$c'(t) = ?$ cost/year

11. Consider the function $r(t)$ representing the flow of water past a bridge, where r is measured in $\frac{m^3}{\text{min}}$ and t is measured in hours since midnight. If $r'(5) = 3$ then:

(A) $3\ m^3$ of water must have flowed past the bridge by 5:00 a.m.

(B) $3\ m^3$ of water must have flowed past the bridge between 5:00 and 6:00 a.m.

(C) ~~water is flowing past the bridge at a rate of $3\frac{m^3}{\text{min}}$ at 5:00 a.m.~~

(D) the volume of water flowing past the bridge is increasing by $3\ \frac{\frac{m^3}{\text{min}}}{hr}$ at 5:00 a.m.

(E) ~~the volume of water flowing past the bridge at 6:00 a.m. will be greater than that which is flowing past the bridge at 5:00 a.m.~~

$r(t) = m^3/\text{min}$

$r'(5) = 3$

hours

$\frac{m^3/\text{min}}{h}$