$$f(x)=\begin{cases}\sin(\pi x), & 0\le x\le 1\\ ax+b, & 1<x\le 2\end{cases}$$

$\pi\cos\pi x\big|_{x=1}$

$a\big|_{x=1}$

$a=-\pi$

f has to be cts @ $x=1$

$$\lim_{x\to1^-}f(x)=\lim_{x\to1^+}f(x)$$

$$\sin(\pi\cdot1)=a\cdot1+b$$

$$\boxed{0=a+b}\qquad\boxed{b=\pi}$$

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

$$\lim_{x\to1^-}\frac{\sin\pi x-0}{x-1}=\lim_{x\to1^+}\frac{ax+b-(a+b)}{x-1}$$

$$\lim_{x\to1^-}\frac{\sin\pi x}{x-1}=\lim_{x\to1^+}\frac{ax-a}{x-1}$$

$$\lim_{x\to1^-}\frac{\pi\cos\pi x}{1}=\lim_{x\to1^+}\frac{a(x-1)}{x-1}$$

$$\boxed{-\pi=a}$$

6. Let $f(x)=x\cdot g(h(x))$, where $g(4)=2, g'(4)=3, h(3)=4$, and $h'(3)=-2$. Find $f'(3)$.

(A) -18
(B) -16
(C) -7
(D) 7
(E) 11

$$f(x)=x\cdot g(h(x))$$

$$f'(x)=g(h(x))+xg'(h(x))\cdot h'(x)$$

$$f'(3)=g(h(3))+3g'(h(3))h'(3)$$

$$=g(4)+3[g'(4)](-2)$$

$$=2+3\cdot3(-2)$$

$$=-16$$

7. If $f(x) = x^2 \ln x$, then $f'(x)$ is

(A) $2x \cdot \ln x$
(B) 2
(C) x
(D) $2x \cdot \ln x + x$
(E) $2x \cdot \ln x + x^2 e^x$

8. If $f(x) = \dfrac{\sin\sqrt{x}}{\sqrt{x}}$, then $f'(x)$ is

(A) $\dfrac{\cos\sqrt{x}}{2x} - \dfrac{\sin\sqrt{x}}{2\sqrt{x^3}}$

(B) $\dfrac{\cos\sqrt{x} - \sin\sqrt{x}}{2x}$

(C) $\dfrac{\sqrt{x}\cos\sqrt{x} - \dfrac{\sin\sqrt{x}}{2\sqrt{x}}}{x}$

(D) $\cos\sqrt{x}$

(E) $\dfrac{\dfrac{\cos\sqrt{x}}{2} + \dfrac{\sin\sqrt{x}}{2\sqrt{x}}}{x}$

9. If $f(x) = \sin^3(x)$, then $f'\left(\dfrac{\pi}{3}\right)$ is

(A) $\dfrac{1}{8}$

(B) $\dfrac{3}{2}$

(C) $\dfrac{9}{4}$

(D) $\dfrac{3\sqrt{3}}{8}$

(E) $\dfrac{9}{8}$

10. A particle moves on the x-axis with position defined by: $x(t) = t^3 - 6t^2 + 2t + 1$; $t \geq 0$. What is the velocity of the particle when its acceleration is zero?

(A) -11
(B) -10
(C) 1
(D) 2
(E) 50

$v(t) = x'(t)$
$a(t) = x''(t) = 0$
$t = \boxed{}$

11. If $f(x) = x^3 + 3x - 1$ and $g(x) = f^{-1}(x)$, find $g'(3)$.
   (A) $\frac{1}{30}$
   (B) $\frac{1}{6}$
   (C) 1
   (D) 6
   (E) 30

$x^3 + 3x - 1 = 3$

$x = \boxed{}$

$g'(3) = \dfrac{1}{f'\boxed{}}$

$f(a) = b$

$g(b) = a$

12. If $x^2y + xy^3 = 8 - 2y$ then the value of $\frac{dy}{dx}$ at (2,1) is
   (A) $-\frac{5}{6}$
   (B) $-\frac{5}{12}$
   (C) 0
   (D) $\frac{5}{12}$
   (E) $\frac{5}{6}$

$2xy + \underline{x^2y'} + 1 \cdot y^3 + \underline{x3y^2y'} = \underline{-2y'}$

13. Which of the following is the equation of the tangent to the function $y = \tan^{-1} 2x$ at $\left(\frac{1}{2}, \frac{\pi}{4}\right)$?
   (A) $y = x + \frac{\pi - 1}{4}$
   (B) $y = \frac{x}{2} + \frac{\pi - 1}{4}$
   (C) $y = x + \frac{\pi - 2}{4}$
   (D) $y = \frac{x}{2} + \frac{\pi - 2}{4}$
   (E) $y = x + \frac{\pi + 1}{4}$

$(\arctan 2x)' = \dfrac{(2x)'}{1 + (2x)^2}$

14. Find $\frac{d^2y}{dx^2}$ if $xy + \pi = 2y^2$.

(A) $\frac{y}{4y-x}$

(B) $\frac{y^2}{(4y-x)^2}$

(C) $\frac{1}{4\left(\frac{y}{4y-x}\right)-x}$

(D) $\frac{-\frac{4y^2}{4y-4}}{(4y-x)^2}$

(E) $\frac{2y-\frac{4y^2}{4y-4}}{(4y-x)^2}$

$1 \cdot y + x \cdot y' = 4yy'$

$y = y'(4y - x)$

$\frac{y}{4y-x} = y'$

$y' = \frac{(4y-x)y' - y(4y'-1)}{(4y-x)^2}$

15. *Which of the following is the equation of the tangent to the curve defined by $x(t) = t^3 + t + 1$ and $y(t) = \tan t$ at $t = 0$?

(A) $y = x - 1$

(B) $y = x$

(C) $y = x + 1$

(D) $y = 1$

(E) None of the above

$m = \frac{dy}{dx} = \frac{dy/dt}{dx/dt}$

$\frac{y'(t)}{x'(t)} = \frac{\sec^2 t}{3t^2+1}$

$y - y_1 = m(x - x_1)$

16. Let $f(x) = \sqrt{1 - \sin x}$

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = 0$.

17. We are given the following information about two differentiable functions f and g.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	4	−3	3	2
3	6	2	−2	3

(a) $(f + g)'(3) =$

(b) $\left(\frac{f}{g}\right)'(1) =$

(c) If $B = f \cdot g$, then $B'(3) =$

(d) $G(x) = \sqrt{f(x)}$, then $G'(1) =$

19. Consider the function $f(x) = x \cdot \ln x$.

(a) Find the instantaneous rate of change of f at $x = e$.

(b) Find the average rate of change of f over the interval $[2.5, 3]$.

20. The velocity of a particle moving along the x-axis is given by the equation $v(t) = \frac{\pi}{4} - \tan^{-1}(t^2 - 4t + 4)$, for $t \geq 0$. The following table gives information about $v(t)$ and $v'(t)$.

t	$0<t<1$	1	$1<t<2$	2	$2<t<3$	3	$t>3$
$v(t)$	Negative	0	Positive	Positive	Positive	0	Negative
$v'(t)$	Positive	Positive	Positive	0	Negative	Negative	Negative

(a) Find an equation for the acceleration of the particle as a function of t. You do not need to simplify the equation.

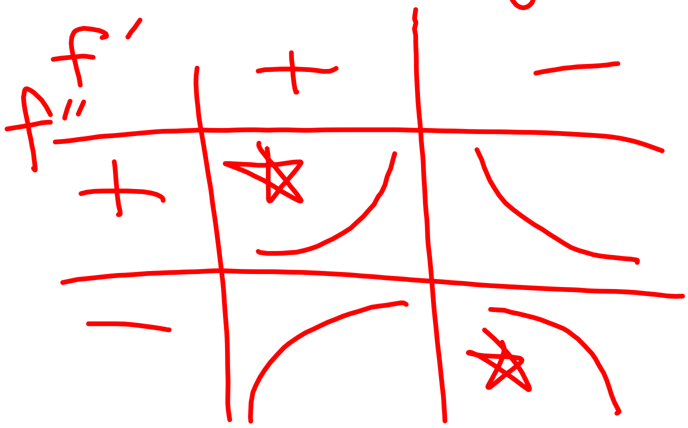
(b) Suppose that on the interval $(0, 2)$, the particle lies in the positive ray of the x-axis. At what times in the interval $(0, 2)$ is the particle moving away from the origin?

$(1, 2)$

(c) During what time intervals is the speed of the particle increasing?

$(1,2) \cup (3, \infty)$

speed = |velocity| > 0



1. Let $f(x) = \sin x$. If the derivative of f at $x = \frac{\pi}{3}$ is one-half the derivative of f at θ then $\theta =$

(A) 0

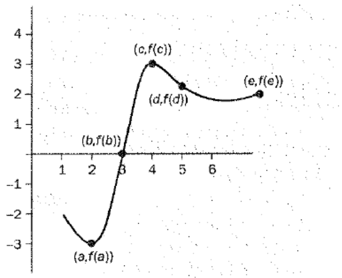
(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

(E) $\frac{\pi}{2}$

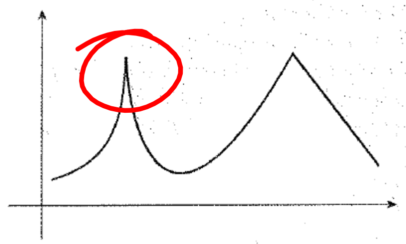
2. The graph of $f(x)$ is shown below with the points A, B, C, D, and E labeled on the graph.



Which of the lists below is presented in ascending order?

(A) $f(b), f'(a), f'(d), f'(b)$
(B) $f(a), f'(c), f'(e), f'(c)$
(C) $f(a), f'(c), f'(b), f'(d)$
(D) $f(a), f'(b), f'(c), f(d)$
(E) $f'(a), f'(c), f(b), f'(d)$

3. The graph of $f(x)$ is shown below.



Which of the following statements is true?

(A) f is differentiable everywhere.
(B) f has two cusps.
(C) f has one cusp.
(D) The graph of f has two vertical tangent lines.
(E) $f'(x)$ is defined everywhere.

cusp = x-value w/ vertical tangent line

4. Suppose $f(x)$ is a differentiable function with $f(1) = 2, f(2) = -2, f'(2) = 5, f'(1) = 3$, and $f(5) = 1$. An equation of a line tangent to the graph of f is

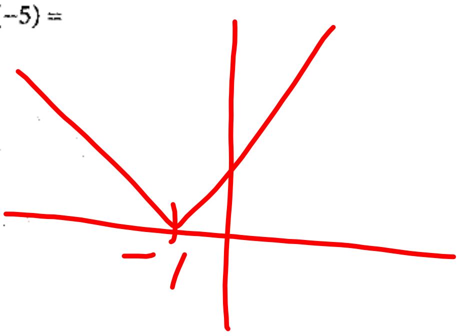
(A) $y - 3 = 2(x - 1)$
(B) $y - 2 = (x - 1)$
(C) $y - 3 = 5(x - 1)$
(D) $y - 2 = 3(x - 1)$
(E) $y - 1 = 5(x - 2)$

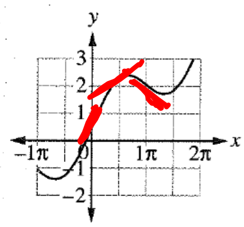
5. Which of the following expressions gives the derivative of $f(x) = \cos 2x$?

(A) $f'(x) = \lim_{x\to 0} \frac{\cos 2x}{2x}$
(B) $f'(x) = \lim_{x\to 0} \frac{\cos 2(x+h) - \cos 2x}{2x}$
(C) $f'(x) = \lim_{h\to 0} \frac{\cos 2(x+h) - \cos 2x}{h}$
(D) $f'(x) = \lim_{h\to 0} \frac{\cos (2x+h) - \cos (2x-h)}{h}$
(E) $f'(x) = \lim_{h\to 0} \frac{\cos 2x - \cos h}{h}$

6. If $f(x) = 4x + 1$, then $f'(-2) =$
   (A) -7
   (B) -2
   (C) 1
   (D) 4
   (E) None of these

7. If $f(x) = |x + 1|$, then $f'(-5) =$
   (A) -5
   (B) -1
   (C) 0
   (D) 1
   (E) 5





8. Consider the graph of $g(x)$ shown above. Which of the following statements is true?
   (A) $g'(0) < g'(\frac{\pi}{2}) < g'(\pi)$
   (B) $g'(\frac{\pi}{2}) < g'(\pi) < g'(0)$
   (C) $g'(\pi) < g'(\frac{\pi}{2}) < g'(0)$
   (D) $g'(\frac{\pi}{2}) < g'(0) < g'(\pi)$
   (E) $g'(0) < g'(\pi) < g'(\frac{\pi}{2})$

9. The function f is given by

$$f(x)=\begin{cases} x \text{ if } -3<x<3 \\ \dfrac{|x|}{x} \text{ if } |x|\geq 3 \end{cases}$$

$$\begin{cases} 1 & x \geq 3 \\ -1 & x \leq -3 \end{cases}$$

Which of the following expressions is largest?

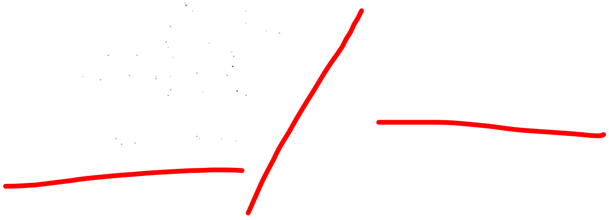
(A) $f(-4)$

(B) $f(0)$

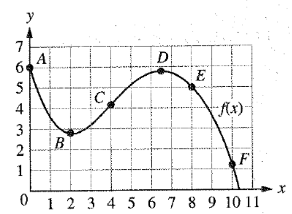
(C) $f'(0)$

(D) $f'(4)$

(E) $f'(5)$



This graph is for problems 10 and 11.



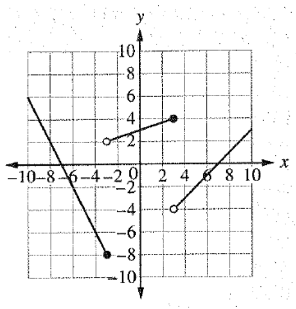
The points labeled A through F lie on the curve $f(x)$.

10. At which point(s) is $f'(x)=0$?

(A) B only
(B) B and D
(C) C and D
(D) E and F
(E) A, E, and F

11. At which point(s) is $f(x)f'(x)<0$?

(A) A only
(B) B only
(C) C and D
(D) E and F only
(E) A, E, and F



The graph of $f(x)$

12. What is the value of $f'(-5) + 2f'(-1) + 3f'(5)$?

(A) $-\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) 1

(E) $\frac{5}{3}$

13. The line $2x - y = 9$ is tangent to the curve $f(x)$ at the point $(4, -1)$. What is the value of $f'(4)$?

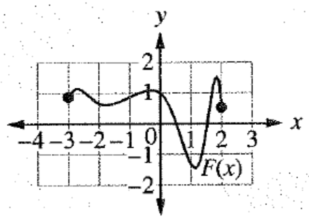
(A) -2

(B) $\frac{1}{2}$

(C) 2

(D) 4

(E) 9



The graph of F(x)

14. At how many distinct points on the curve is $F'(x) = 0$?

(A) 2
(B) 3
(C) 4
(D) 5
(E) 6

15. If $f(x) = e^x$, which of the following is equal to $f'(e)$?

(A) $\lim\limits_{h\to 0} \frac{e^{x+h}}{h}$

(B) $\lim\limits_{h\to 0} \frac{e^{x+h} - e^e}{h}$

(C) $\lim\limits_{h\to 0} \frac{e^{e+h} - e}{h}$

(D) $\lim\limits_{h\to 0} \frac{e^{x+h} - 1}{h}$

(E) $\lim\limits_{h\to 0} \frac{e^{e+h} - e^e}{h}$

17. You may use a calculator for this question.

During the course of a 15-hour storm, the water levels of a reservoir are measured. In addition, some data about the rate of change of the water level is collected. The data is summarized in the table below. Assume that h and h' are both continuous and differentiable functions of t for $0 \le t \le 15$.

Time, t (hours)	2	3	8	12	15
Water level $h(t)$ (feet)	428	432	457	477	483
Rate of change $h'(t)$ (feet/hour)	**	$3.5\ \frac{\text{ft}}{\text{h}}$	$4.3\ \frac{\text{ft}}{\text{h}}$	**	$6.4\ \frac{\text{ft}}{\text{h}}$

(a) Write the equation for the tangent line to the graph of h at $t = 3$.

(b) Compute the average rate of change over the interval $[2,15]$. Using the Intermediate Value Theorem and the data for $h'(t)$ determine in which time intervals there must be a time where the instantaneous rate of change is equal to the average rate of change.

(c) Is the data collected about the rate of change of the water level, $h'(t)$, consistent with the statement that $h''(t) > 0$ on the interval $2 < t < 15$? Explain your answer.

(d) Is the data collected about the water level, $h(t)$, consistent with the statement that $h''(t) > 0$ on the interval $0 \le t \le 15$?Explain your answer.

18. A table of historical national population estimates in the United States from 1990 to 1999 is shown below.

Historical National Population Estimates: 1990–1999*

Year	National Population	Population Change
1999	272,690,813	2,442,810
1998	270,248,003	2,464,396
1997	267,783,607	2,555,035
1996	265,228,572	2,425,296
1995	262,803,276	2,476,255
1994	260,327,021	2,544,413
1993	257,782,608	2,752,909
1992	255,029,699	2,876,607
1991	252,153,092	2,688,696
1990	249,464,396	2,645,166

* Source: Population Estimates Program, Population Division, U.S. Census

Bureau Internet Release Date: April 11, 2000
Revised date: June 28, 2000 http://www.census.gov/popest/data/intercensal/national/index.html

(a) Use the data on the table to approximate the rate of change of the U.S. population in 1999. Show the computations that lead to your answer. Indicate the units of measure.

(b) Use your answer from part (a) to estimate the U.S. population in the year 2000. Show the computations that lead to your answer.

(a) $\frac{\Delta P}{\Delta t} = \frac{2442810}{1}$

$\approx P'(1999)$

$P'(1999) \approx \frac{P(2000) - P(1999)}{2000 - 1999}$

$P'(1999) \cdot 1 + P(1999) \approx P(2000)$