right-hand sum

$$\sum_{i=1}^{n} \underbrace{\frac{b-a}{n}}_{\Delta x} f\left(a+\frac{b-a}{n}i\right)$$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{b-a}{n} f\left(a+\frac{b-a}{n}i\right)$$

Fundamental Theorem of Calculus

If F is an antiderivative of f,

then $\int_a^b f(x)\,dx = F(b)-F(a)$

MVT for derivatives:

If f is cts on $[a,b]$ & diff. on (a,b),

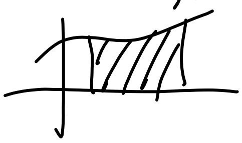
$\exists\, c \in (a,b)$ s.t.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$(b-a)f'(c) = f(b)-f(a)$$

The average value of f on (a,b)

is $\frac{1}{b-a}\int_a^b f(x)\,dx$



MVT for integrals

$$\int_a^b f(x)\,dx = f(c)\cdot(b-a)$$

2nd Fundamental Theorem of Calculus

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$$\frac{d}{dx}\int_a^{g(x)} f(t)\,dt = f(g(x)) \cdot g'(x)$$

Let $g(x) = \int_2^{3x^2} \cos t\,dt$

Find $g'(x) = \frac{dg}{dx} = \cos(3x^2) \cdot 6x$

Let $g(x) = \int_{2x}^{\cos x} 5t^3\,dt$

$$= \int_{2x}^{a} + \int_{a}^{\cos x} = \int_{a}^{\cos x} 5t^3\,dt - \int_{a}^{2x} 5t^3\,dt$$

$$g'(t) = 5(\cos x)^3 \cdot (-\sin x) - 5(2x)^3 \cdot 2$$

Find the average value of $\sec^2 x$ on $[0, \pi/4]$

$$\frac{\sec^2 \pi/4 - \sec^2 0}{\pi/4 - 0}$$ = average rate of change of $\sec^2 x$ on $[0, \pi/4]$

average value:

$$\frac{1}{\pi/4 - 0} \int_0^{\pi/4} \sec^2 x\,dx$$

$$= \frac{1}{\pi/4} \tan x \Big|_0^{\pi/4} = \frac{4}{\pi}\left(\tan\frac{\pi}{4} - \tan 0\right)$$

$$= \frac{4}{\pi}(1 - 0)$$

$$= \boxed{\frac{4}{\pi}}$$

$$\int x^2 \sec^2 x^3 \, \cancel{dx} = \int \frac{1}{3}\sec^2 u \, du$$

$u = x^3$

$du = 3x^2 \, dx$

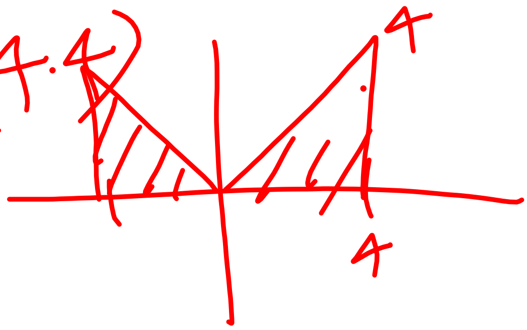
$\frac{1}{3} du = x^2 \, dx$

$$= \frac{1}{3}\tan u + C$$

$$= \frac{1}{3}\tan\left(x^3\right) + C$$

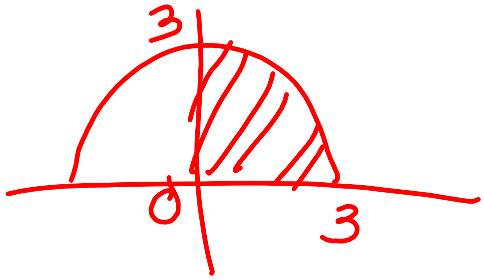
$$\int_{-4}^{4} |x| \, dx = 2\left(\frac{1}{2} 4 \cdot 4\right)$$

$$= 16$$



$$\int_{-4}^{0} -x \, dx + \int_{0}^{4} x \, dx$$

$$\int_0^3 \sqrt{9-x^2}\,dx$$



$$\frac{1}{4}\pi(3)^2$$

$$(x,y)=(r\cos\theta, r\sin\theta)$$

$$x^2+y^2=r^2$$

$$y=\pm\sqrt{r^2-x}$$

$$=\pm r\sqrt{1-\cos^2\theta}$$

$$=\pm r\sin\theta$$

$$\int \frac{\cos x}{1-\sin x}\,dx = \int -\frac{du}{u} = -\ln|u|$$

$$= -\ln|1-\sin x|$$

$$= \ln\frac{1}{|1-\sin x|}+C$$

$u=1-\sin x$

$du=-\cos x\,dx$

$-du=\cos x\,dx$

$$\int \frac{e^x+e^{-x}}{e^x-e^{-x}}\,dx = \int \frac{du}{u} = \ln|e^x-e^{-x}|+C$$

$u = e^x - e^{-x}$

$du = (e^x + e^{-x})\,dx$

$$\int x4^{-x^2}\,dx = \int -\frac{1}{2}\cdot 4^u\,du$$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2}du = x\,dx$

$$= \frac{-\frac{1}{2}\,4^u}{\ln 4}+C$$

$$= \frac{-4^{-x^2}}{2\ln 4}+C$$

$$\int \sin^4 x\,dx = \int \sin^2 x \sin^2 x\,dx$$

$\cos 2\theta = 1 - 2\sin^2\theta$ $\qquad = \int (1-\cos^2 x)\sin^2 x\,dx$

$\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\int (\sin^2 x)^2\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 dx = \frac{1}{4}\int (1-2\cos 2x+\cos^2 2x$$

$$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\,dx$$

$$= \frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{4}\int \cos^2 2x\,dx$$

$\cos 2\theta = 2\cos^2\theta - 1$ $\qquad \dfrac{1}{4}\displaystyle\int \frac{\cos 4\theta + 1}{2}\,dx$

$\dfrac{\cos 2\theta + 1}{2} = \cos^2\theta$

$$= \boxed{\frac{1}{4}x - \frac{1}{4}\sin 2x + \frac{1}{8}x + \frac{1}{32}\sin 4\theta + C}$$