$$\int x^2 \ln x \, dx \qquad \int u\,dv = uv - \int v\,du$$

$$u = \ln x \qquad dv = x^2\,dx$$

$$du = \frac{dx}{x} \qquad v = \frac{1}{3}x^3$$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \cdot \frac{dx}{x}$$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\,dx$$

$$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + C$$

$$\int e^x \sin x \, dx$$

$$u = \sin x \qquad dv = e^x\,dx$$

$$du = \cos x\,dx \qquad v = e^x$$

$$= e^x \sin x - \int e^x \cos x \, dx$$

$$u = \cos x \qquad dv = e^x\,dx$$

$$du = -\sin x\,dx \qquad v = e^x$$

$$\int e^x \sin x\,dx = e^x \sin x - \left(e^x \cos x - \int -e^x \sin x\,dx\right)$$

$$\int e^x \sin x\,dx = e^x \sin x - e^x \cos x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x\,dx = \frac{e^x \sin x - e^x \cos x}{2} + C$$

$$\int \frac{\sec^2 x\,dx}{\sqrt{1-\tan^2 x}} = \int \frac{du}{\sqrt{1-u^2}} = \arcsin u + c$$

$u = \tan x$

$du = \sec^2 x\,dx$

$$= \arcsin(\tan x) + c$$

$$\int \frac{e^{3x}\,dx}{1+e^{6x}} = \int \frac{\frac{1}{3}du}{1+u^2} = \frac{1}{3}\arctan u + c$$

$u = e^{3x}$

$du = 3e^{3x}\,dx$

$\frac{1}{3}du = e^{3x}\,dx$

$$= \frac{1}{3}\arctan(e^{3x}) + c$$

Find the dimensions of the rectangle of maximum area bounded by the x-axis and the graph of $y=4-x^2$

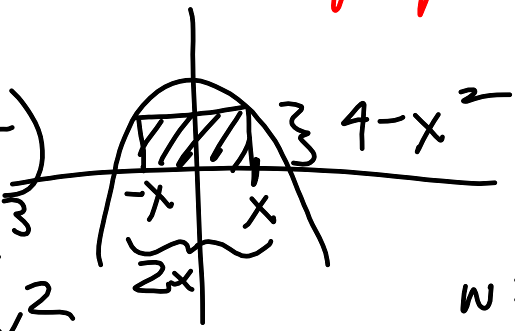
$A = bh$

$A(x) = 2x(4-x^2)$

$A(x) = 8x - 2x^3$

$A'(x) = 8 - 6x^2$

$8 = 6x^2$



$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

$w = \frac{4}{\sqrt{3}}$

$h = 4 - \frac{4}{3}$

$= \frac{8}{3}$