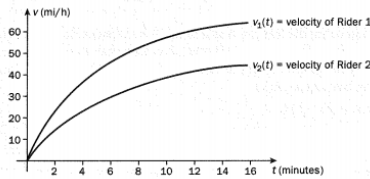


17. The graph below shows the velocity of two bicyclists 1 and 2 in a race from time  $t = 0$  to time  $t = 16$ . The racers start side by side and travel along the same road. The race begins at time  $t = 0$ . At time  $t = 16$  one of the racers completes the course.



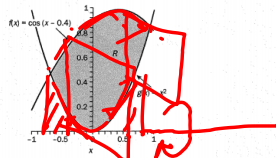
- (a) What does the area between the two curves represent?
- (b) Which bicyclist finishes the course at time  $t = 16$ ?
- (c) Assuming that the velocity of the other racer continues as an integrable function beyond the portion shown on the graph, set up (but do not solve) an equation to find the time at which the second racer finishes the course.

(a) The area under a time v. velocity graph is total distance traveled so, the area between these two curves represents the distance between the two rider at time  $t = 16$ .

(b) Rider 1 finishes first because both velocities are positive and increasing on  $(0, 16)$  and  $v_1(t) > v_2(t)$  on  $(0, 16)$ . At any given time  $t$  Rider 1 has traveled further than Rider 2.

(c)  $\int_0^x v_1(t) dt = \int_0^x v_2(t) dt$   
 where  $x$  is the time in minutes when Rider 2 finishes the course.

18. Consider the region  $R$  enclosed by the graphs of  $f(x) = \cos(x - 0.4)$  and  $g(x) = x^2$ , shown below.



- (a) Find the points of intersection of the graphs of  $f$  and  $g$ .
- (b) Find the area of the region  $R$ .
- (c) Find the volume generated by revolving the region  $R$  about the line  $y = 0$ .
- (d) The region  $R$  is the base of a solid; each cross section of the solid, in a plane perpendicular to the  $x$ -axis, is a rectangle, the shorter side of which lies in the base and the longer side of which is three times its width. Find the volume of the solid.

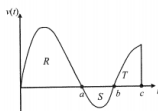
(a)  $\cos(x - 0.4) = x^2$   
 solve  $(\cos(x - 0.4) = x^2, x)$   
 $x = -0.683974$   
 $x = 0.929115$

(b)  $\int_{-0.684}^{0.929} (\cos(x - 0.4) - x^2) dx$   
 $= \int_{-0.684}^{0.929} (\cos(x - 0.4) - x^2, x, -0.684, 0.929)$   
 $= 1.014$

(c)  $\int_{-0.684}^{0.929} \pi (\cos(x - 0.4))^2 dx - \int_{-0.684}^{0.929} \pi (x^2)^2 dx$   
 $= \pi \int_{-0.684}^{0.929} (\cos(x - 0.4))^2 - x^4 dx$   
 $= 3.339$

(d)  $V = \int_{-0.684}^{0.929} 3(\cos(x - 0.4) - x^2)^2 dx$   
 cross-sectional area  
 $= 2.302$

19. Let  $f$  be the function given by  $f(x) = \sqrt{x+3}$  and  $R$  be the region enclosed by the graph of  $f$ , the  $x$ -axis, and the vertical line  $x = 6$ .
- Sketch a graph of the enclosed region  $R$ .
  - Write an integral expression that equals the exact area of the region  $R$ . Evaluate the integral.
  - Rather than using the line  $x = 6$  as a boundary, consider the line  $x = k$  where  $k$  is a number greater than  $-3$ . Let  $A(k)$  be a function that gives the area of the region enclosed by the graph of  $f$ , the  $x$ -axis, and the vertical line  $x = k$ . Write an integral expression that equals  $A(k)$ .
  - Solve for  $k$ :  $A(k) = 144$ .



The above graph shows the velocity of a particle moving along the  $y$ -axis for  $0 \leq t \leq c$ . The particle is at the origin at  $t = 0$ . The region  $R$  has area 8, the region  $S$  has area 1.3, and the region  $T$  has area 1.7.

- What is the position of the particle at time  $t = a$ ? At  $t = b$ ? At  $t = c$ ?
- At what time  $t$  is the particle farthest from the origin?
- What is the total distance traveled by the particle?

$$y = (x^2 + 1)^x$$
$$\ln y = \ln (x^2 + 1)^x$$
$$\ln y = x \ln (x^2 + 1)$$
$$\frac{1}{y} \cdot y' = \underline{\hspace{2cm}}$$
$$y' = \underline{\hspace{2cm}} \cdot y$$