1. If $\frac{dy}{dx} = \frac{7x^2}{y^3}$ and $y(3) = 2$, find an equation for y in terms of x.

$\int y^3 dy = \int 7x^2 dx$

$\frac{1}{4}y^4 = \frac{7}{3}x^3 + C$ ← general solution

$\frac{1}{4}(2)^4 = \frac{7}{3}(3)^3 + C$

$4 = 63 + C$

$-59 = C$

particular solution

$\frac{1}{4}y^4 = \frac{7}{3}x^3 - 59$

2. If $\frac{dy}{dx} = 5x^2 y$ and $y(0) = 6$, find an equation for y in terms of x.

$\int \frac{dy}{y} = \int 5x^2 dx$

$\ln|y| = \frac{5}{3}x^3 + C$

$\ln|6| = \frac{5}{3}(0)^3 + C$

$\ln 6 = C$

$\ln|y| = \frac{5}{3}x^3 + \ln 6$

$e^{\ln|y|} = e^{\frac{5}{3}x^3 + \ln 6}$

$|y| = e^{5/3 x^3 + \ln 6}$

$|y| = e^{5/3 x^3} e^{\ln 6}$

$|y| = 6e^{5/3 x^3}$

$y = \pm 6e^{5/3 x^3}$

3. If $\frac{dy}{dx} = \frac{1}{y + x^2 y}$ and $y(0) = 2$, find an equation for y in terms of x.

$$\frac{dy}{dx} = \frac{1}{y(1+x^2)}$$

$$\int y\,dy = \int \frac{dx}{1+x^2}$$

$$\frac{1}{2}y^2 = \arctan x + C$$

$$\frac{1}{2}(2)^2 = \arctan 0 + C$$

$$2 = 0 + C$$

$$2 = C$$

$$\frac{1}{2}y^2 = \arctan x + 2$$

$$y^2 = 2\arctan x + 4$$

$$y = \pm\sqrt{2\arctan x + 4}$$

4. If $\frac{dy}{dx} = \frac{e^x}{y^2}$ and $y(0) = 1$, find an equation for y in terms of x.

$$\int y^2\,dy = \int e^x\,dx$$

$$\frac{1}{3}y^3 = e^x + C$$

$$\frac{1}{3}(1)^3 = e^0 + C$$

$$\frac{1}{3} = 1 + C$$

$$-\frac{2}{3} = C$$

$$\frac{1}{3}y^3 = e^x - \frac{2}{3}$$

$$y^3 = 3e^x - 2$$

$$y = \sqrt[3]{3e^x - 2}$$

5. If $\frac{dy}{dx} = \frac{y^2}{x^3}$ and $y(1) = 2$, find an equation for y in terms of x.

$$\frac{dy}{y^2} = \frac{dx}{x^3}$$

$$\int y^{-2}\,dy = \int x^{-3}\,dx$$

$$-y^{-1} = -\frac{1}{2}x^{-2} + C$$

$$\frac{-1}{y} = \frac{-1}{2x^2} + C$$

$$y(1) = 2$$

$$-\frac{1}{2} = \frac{-1}{2(1)^2} + C$$

$$0 = C$$

$$\frac{-1}{y} = \frac{-1}{2x^2}$$

$$y = 2x^2$$

6. If $\frac{dy}{dx} = \frac{\sin x}{\cos y}$ and $y(0) = \frac{3\pi}{2}$, find an equation for y in terms of x.

$$\int \cos y\,dy = \int \sin x\,dx$$

$$\sin y = -\cos x + C$$

$$\sin\frac{3\pi}{2} = -\cos 0 + C$$

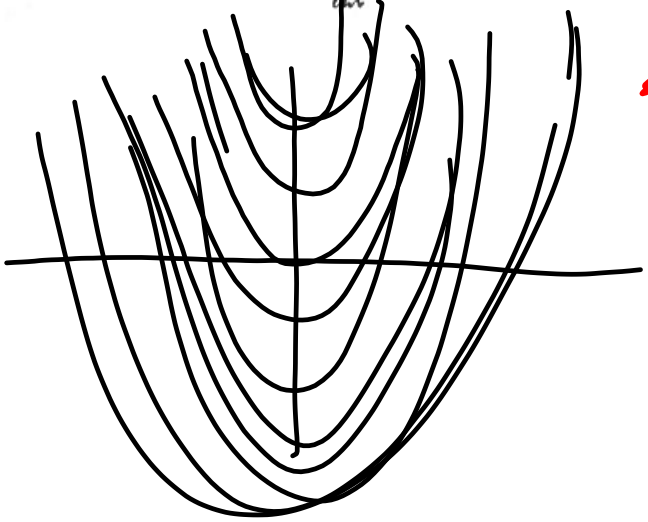
$$-1 = -1 + C$$

$$0 = C$$

$$\sin y = -\cos x$$

$$y = \arcsin(-\cos x)$$

14. Sketch the slope field for $\frac{dy}{dx} = 2x$.

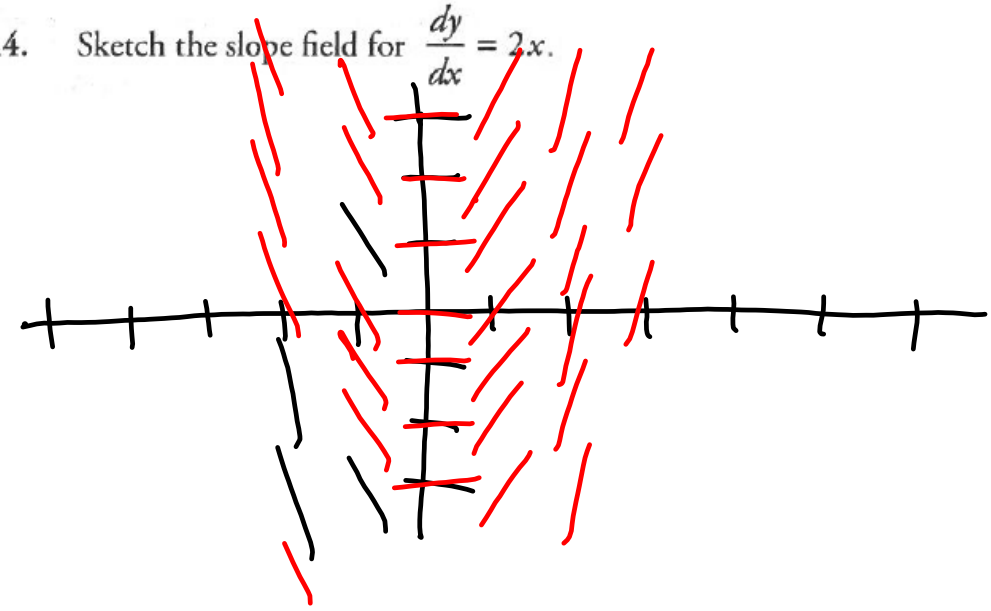


not the slope field

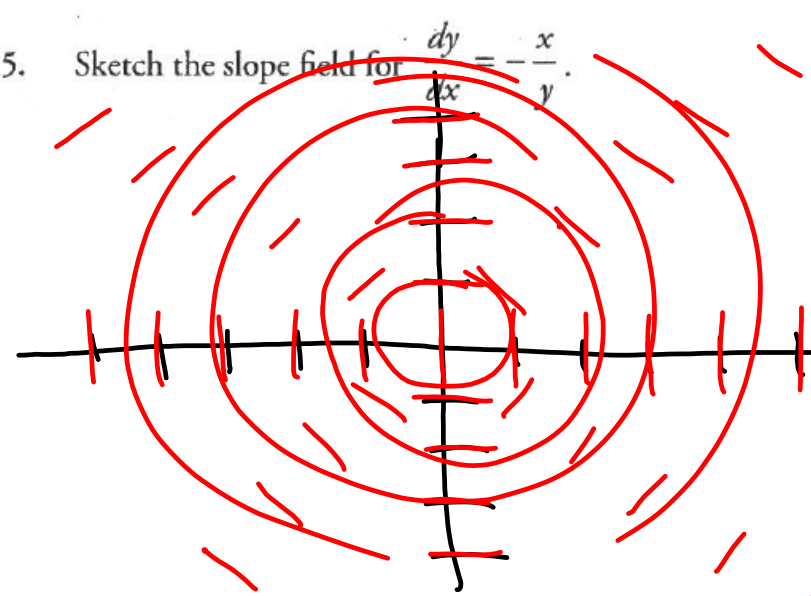
$dy = 2x\,dx$

$y = x^2 + C$

14. Sketch the slope field for $\frac{dy}{dx} = 2x$.



15. Sketch the slope field for $\frac{dy}{dx} = -\frac{x}{y}$.



$y\,dy = -x\,dx$

$\frac{1}{2}y^2 = -\frac{1}{2}x^2 + c$

$y^2 = -x^2 + C$

$y^2 = C - x^2$

$x^2 + y^2 = C$

16. Sketch the slope field for $\frac{dy}{dx} = \frac{x}{y}$.