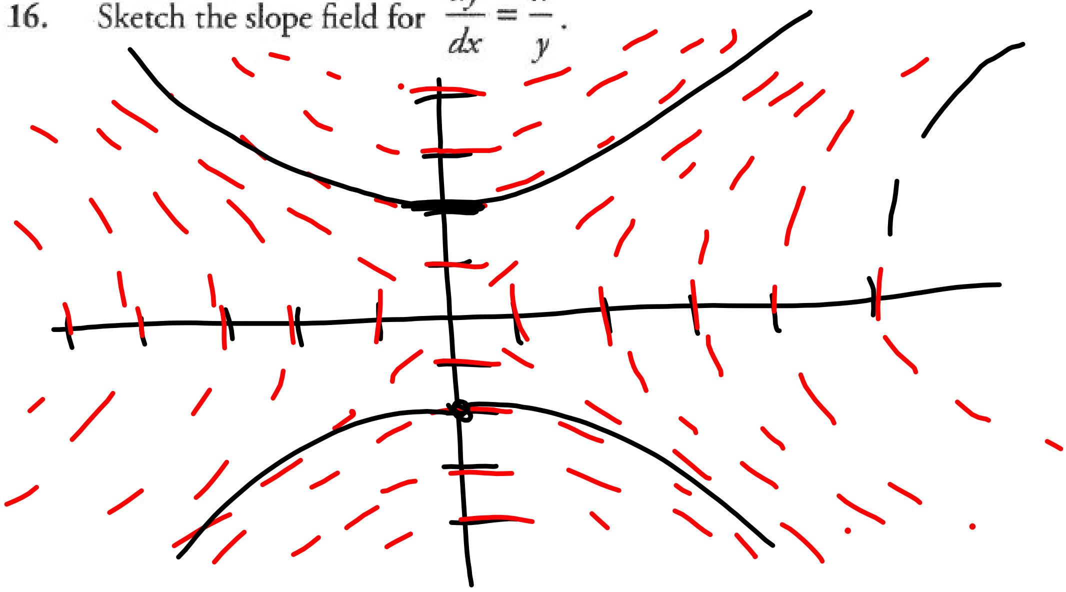
16. Sketch the slope field for $\frac{dy}{dx} = \frac{x}{y}$.



$$\frac{dy}{dx} = k \cdot y$$

Rate of change of a function is proportional to the function value.

$$\frac{dy}{y} = k\,dx$$

$$\ln|y| = kx + C$$

$$e^{\ln|y|} = e^{kx+C}$$

$$|y| = e^{c} e^{kx}$$

$$y = \pm e^{c} e^{kx} \longrightarrow y = Ae^{kx}$$

$$y(0) = Ae^{k \cdot 0}$$

$$y(0) = A$$

$$P(t) = P_0 e^{kt}$$

A colony of bacteria grows exponentially and the colony's population is 4,000 at time $t = 0$ and 6,500 at time $t = 3$. How big is the population at time $t = 10$?

$P(0) = 4000$
$P(3) = 6500$
$P(10) = ?$

$P(t) = P_0 e^{Kt}$

$6500 = 4000 e^{K \cdot 3}$

$\frac{6500}{4000} = e^{K \cdot 3}$

$\ln\left(\frac{65}{40}\right) = K \cdot 3$

$\frac{\ln\left(\frac{65}{40}\right)}{3} = K$

$P(10) = 4000 e^{\frac{\ln\left(\frac{65}{40}\right)}{3} \cdot 10}$

≈ 20179.228

A rock is thrown upward with an initial velocity, $v(t)$, of 18 m/s from a height, $h(t)$, of 45 m. If the acceleration of the rock is a constant -9 m/s^2, find the height of the rock at time $t = 4$.

$v(t) = h'(t)$
$a(t) = v'(t)$
$a(t) = -9$
$v(t) = -9t + v_0 = -9t + 18$
$h(t) = -\frac{9}{2}t^2 + v_0 t + h_0$
$= -\frac{9}{2}t^2 + 18t + 45$
$h(4) = -\frac{9}{2}(4)^2 + 18(4) + 45$
$= 45\text{ m}$

The rate of growth of the volume of a sphere is proportional to its volume. If the volume of the sphere is initially 36π ft^3, and expands to 90π ft^3 after 1 second, find the volume of the sphere after 3 seconds.

$$\frac{dV}{dt} = kV \rightarrow V(t) = V_0 e^{kt}$$

$$90\pi = 36\pi e^{k\cdot 1}$$

$$\frac{5}{2} = e^{k}$$

$$\ln\frac{5}{2} = k$$

$$V(t) = 36\pi e^{\ln\frac{5}{2} t}$$

$$V(3) = 36\pi e^{\ln\frac{5}{2}\cdot 3}$$

$$\approx 1767.146$$

A radioactive element decays exponentially in proportion to its mass. One-half of its original amount remains after 5,750 years. If 10,000 grams of the element are present initially, how much will be left after 1,000 years?

$$\frac{dM}{dt} = k\cdot M$$

$$M(t) = M_0 e^{kt}$$

$$\frac{1}{2}\cancel{M_0} = \cancel{M_0} e^{k\cdot 5750}$$

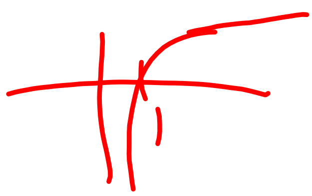
$$\ln\frac{1}{2} = k\cdot 5750$$

$$\frac{\ln\frac{1}{2}}{5750} = k$$

$$M(t) = 10000 e^{\frac{\ln\frac{1}{2}}{5750} t}$$

$$M(1000) =$$

$$\approx 8864.351$$



1. A particle moves along the x-axis with a velocity given by $v(t) = 2 + \sin t$. When $t = 0$ the particle is at $x = -2$. Where is the particle when $t = \pi$?

(A) π
$\boxed{\text{(B) } 2\pi}$
(C) $\pi - 1$
(D) $\pi - 2$
(E) $\pi + 1$

$x(t) = 2t - \cos t + C$

$-2 = 2(0) - \cos(0) + c$

$-2 = -1 + c$

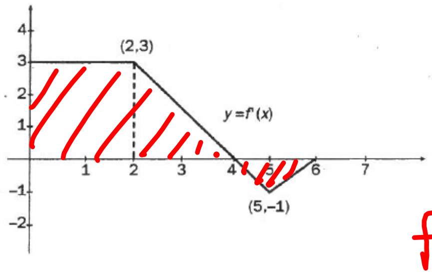
$-1 = c$

$x(\pi) = 2\pi - \cos\pi - 1 = 2\pi - (-1) - 1 = 2\pi$

<u>1st Fundamental Theorem of Calculus</u>:
If a function f is continuous on the closed interval $[a, b]$ and F is an antiderivative of f on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

3. The graph of the derivative of f, f', is shown below.



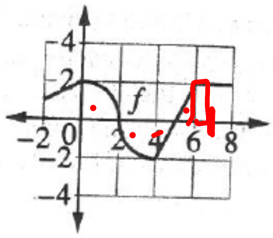
If $f(0) = 7$, find $f(6)$.
(A) 9
(B) 11
(C) 12
(D) 14
$\boxed{\text{(E) } 15}$

$\int_0^6 f'(x)\,dx = f(6) - f(0)$

$f(0) + \int_0^6 f'(x)\,dx = f(6)$

$7 + 2(3) + \frac{1}{2}(2)(3) - \frac{1}{2}(2)(1) = f(6)$

$7 + 6 + 3 - 1$



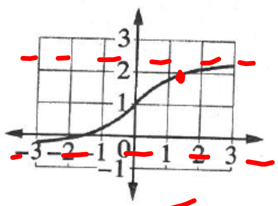
7. Given the graph of f above, if $F'(x) = f(x)$ and $F(0) = 3$, what is $F(7)$?

(A) 2
(B) 3
$\boxed{\text{(C) 5}}$
(D) $2\pi + 5$
(E) $2\pi + 7$

$\int_0^7 = F(7) - F(0)$

$F(7) = 3 + \int_0^7 = 3 + 1 \cdot 2$
$= 5$

12. The graph of the function shown to the right is a solution to one of the differential equations below. Which one?



~~(A) $y' = 1 - x^2y^2$~~
~~(B) $y' = 1 + x^2y^2$~~
(C) $y' = \frac{1}{1+y^2}$
~~(D) $y' = \frac{1}{1-x^2}$~~
(E) $y' = \frac{1}{1+x^2}$

$(\frac{3}{2}, 2)$

(B) $y' = 1 + (\frac{3}{2})^2(2)^2$
$= 10$

$\frac{dy}{dx} = \frac{1}{1+y^2} \rightarrow (1+y^2)\,dy = dx$

$y + \frac{1}{3}y^3 + C = x$

$\frac{dy}{dx} = \frac{1}{1+x^2} \rightarrow dy = \frac{dx}{1+x^2}$

$y = \arctan x + C$