1. C
2. A
3. C
4. B
5. A
6. C
7. C
8. E
9. D
10. C
11. A
12. A
13. D
14. E
15. E

$$-\ln|3-y| = \sin x - \ln 2$$

$$\cancel{3-y = -e^{\sin x - \ln 2}}$$

$$\ln|3-y| = \ln 2 - \sin x$$

$$|3-y| = e^{\ln 2 - \sin x}$$

$$\frac{dB}{dt} = \frac{1}{5}(100-B)$$

if $B < 100$

$$\frac{d^2B}{dt^2} < 0$$

& concave down

$$\frac{dB}{dt} = 20 - \frac{1}{5}B$$

$$\frac{d^2B}{dt^2} = -\frac{1}{5} \cdot \frac{dB}{dt} = -\frac{1}{5}\left(\frac{1}{5}(100-B)\right)$$

$$= -\frac{1}{25}(100-B)$$

An ***infinite series*** (or simply a ***series***) is an infinite sum $a_1 + a_2 + \ldots$. This sum is often written using "sigma" notation:

$$a_1 + a_2 + \ldots + a_n + \ldots = \sum_{n=1}^{\infty} a_n$$

The sum of a series is defined as the limit of a sequence, namely the limit of the sequence of the ***partial sums***. The sums

$$s_1 = a_1$$
$$s_2 = a_1 + a_2$$
$$\ldots$$
$$s_n = a_1 + a_2 + \ldots + a_n$$
$$\ldots$$

are called ***partial sums*** for the series $\sum_{n=1}^{\infty} a_n$.

We say that L is the limit of a sequence $s_1, s_2, s_3, \ldots$ and write $\lim_{n\to\infty} s_n = L$ if, for any sufficiently large n, s_n is close to L.[*]

If the limit of the partial sums $\lim_{n\to\infty} s_n = S$ exists, we say that the infinite series $\sum_{n=1}^{\infty} a_n$ *converges* and call S the sum of the series. Otherwise we say that the series *diverges*.

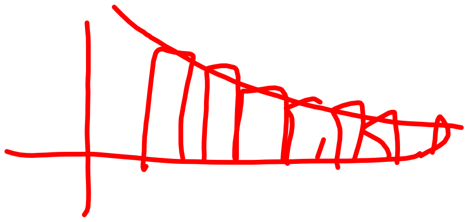
The geometric series $\sum_{i=1}^{\infty} ar^{i-1}$ converges if and only if $|r| < 1$. If the series converges, its sum is $\frac{a}{1-r}$.

$\frac{1}{n}$ **just because the n-th term goes to zero does not mean the series necessarily converges.**

harmonic ser. is: $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$

n-th Term Test	$\sum_{n=1}^{\infty} a_n$ converges $\Rightarrow \lim_{n\to\infty} a_n = 0$. $\lim_{n\to\infty} a_n \neq 0 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges.
Integral Test	$f(x)$ is continuous, positive, and decreasing. $\sum_{n=1}^{\infty} f(n)$ converges $\Leftrightarrow \int_M^{\infty} f(x)\,dx$ converges (for some M).
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges $\Leftrightarrow p > 1$.
Comparison Test	$0 < a_n < b_n$. $\sum_{n=1}^{\infty} b_n$ converges $\Rightarrow \sum_{n=1}^{\infty} a_n$ converges. $\sum_{n=1}^{\infty} a_n$ diverges $\Rightarrow \sum_{n=1}^{\infty} b_n$ diverges.
Ratio Test	$\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert < 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ converges. $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert > 1 \Rightarrow \sum_{n=1}^{\infty} a_n$ diverges. $\lim_{n\to\infty} \left\lvert\frac{a_{n+1}}{a_n}\right\rvert = 1 \Rightarrow$ can't tell.
Alternating Series Test	$a_n > 0$, decreasing, $\lim_{n\to\infty} a_n = 0 \Rightarrow \sum_{n=1}^{\infty} (-1)^{n-1} a_n$ converges.

Table 9-1. Series Convergence Tests



Which is NOT absolutely convergent?

(A) $\sum_{n=1}^{\infty} (-1)^n \frac{n^5}{5^n}$ ratio test $\frac{(n+1)^5}{5^{n+1}} \cdot \frac{5^n}{n^5}$ $\quad \frac{1}{5} \cdot \frac{(n+1)^5}{n^5}$

(D) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ p-series

(B) $\sum_{n=1}^{\infty} (-5)^n \frac{1}{\sqrt[3]{n}}$ diverges by ratio test $\frac{5^{n+1}}{\sqrt[3]{n+1}} \cdot \frac{\sqrt[3]{n}}{5^n}$ $\quad 5 \cdot \frac{\sqrt[3]{n}}{\sqrt[3]{n+1}} = 5$

(E) all are absolutely convergent

(C) $\sum_{n=1}^{\infty} \frac{\cos 3n}{n^3}$ comparison w/ p-series

$0 \quad \cancel{\frac{-1}{n^3}} \le \left\lvert \frac{\cos 3n}{n^3} \right\rvert \le \left\lvert \frac{1}{n^3} \right\rvert$

Ratio Test inconclusive?

(A) $\sum_{n=1}^{\infty} \frac{5^{n+1}}{n^3}$ $\quad \frac{5^{n+1}}{(n+1)^3} \cdot \frac{n^3}{5^n} = 5$

(B) $\sum_{n=1}^{\infty} \frac{n}{2^n}$ $\quad \frac{n+1}{2^{n+1}} \cdot \frac{2^n}{n} = \frac{1}{2}$

(C) $\sum_{n=1}^{\infty} \frac{(-3)^{n-1}}{\sqrt{n}}$ $\quad \frac{(-3)^n}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{-3^{n-1}} = -3$

(D) $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{1+n^2}$ $\quad \frac{\sqrt{n+1}}{1+(n+1)^2} \cdot \frac{1+n^2}{\sqrt{n}} = \frac{n^2\sqrt{n}}{n^2\sqrt{n}} = 1$

(E) $\sum_{n=1}^{\infty} \frac{n^n}{n!}$ $\quad \frac{(n+1)^{n+1}}{(n+1)!} \cdot \frac{n!}{n^n}$

$$\frac{(n+1)^{n+1}}{n+1 \cdot n^n} = \frac{(n+1)^n}{n^n}$$

$$\text{Let } y = \lim_{n\to\infty}\left(\frac{n+1}{n}\right)^n$$

$$\ln y = \lim_{n\to\infty} n \ln\frac{n+1}{n}$$

$$\ln y = \lim_{n\to\infty} \frac{\ln\frac{n+1}{n}}{\frac{1}{n}} = \lim_{n\to\infty} \frac{\frac{n}{n+1}\cdot\frac{n\cdot 1-(n+1)}{\cancel{n^2}}}{\frac{-1}{\cancel{n^2}}} = \lim_{n\to\infty} \frac{+n}{n+1} = 1$$

$$\ln y = 1$$

$$y = e$$