$$\int_{n+1}^{\infty} f(x)\,dx \le \underset{E_n}{R_n} \le \int_{n}^{\infty} f(x)\,dx$$

$3x^{-2}$

$$R_n \le \int_{n}^{\infty} \frac{3}{x^2}\,dx = \lim_{t\to\infty} \int_{n}^{t} \frac{3}{x^2}\,dx$$

$$= \lim_{t\to\infty} -\frac{3}{x}\Big|_n^t = \lim_{t\to\infty} \frac{-3}{t} - \left(\frac{-3}{n}\right) = \frac{3}{n}$$

$$\frac{3}{n} < 0.001$$

$$\frac{n}{3} > \frac{1}{0.001}$$

$$n > \frac{3}{0.001} = 3000$$

$$R_K(x) = f(x) - f_K(x)$$

$$f_K = \sum_{n=0}^{K} \frac{x^n}{n!} \qquad \lim_{K\to\infty} f_K(x) = f(x)$$

$$\lim_{k\to\infty} R_k(x) = 0 \quad \text{for } |x-a| < R$$

$$\Rightarrow f(x) = \sum_{n=0}^{\infty} \frac{f^n(a)}{n!}(x-a)^n \text{ for all}$$

x such that $|x-a| < R$.

suppose $\left|f^{(k+1)}(x)\right| \le M$ when $|x-a| \le R$,

then $R_k(x) \le \frac{M}{(k+1)!}|x-a|^{k+1}$ for all x such that $|x-a| \le R$

$$\frac{0.7}{4!}|3.6-3|^4$$

$$\sum_{n=0}^{\infty} \frac{n x^{2n+3}}{x^2 \cdot n!}$$

$2n+5$

$2(n+1)+3$

$$\frac{\cancel{(n+1)}\, x^{2(n+1)+3}}{\cancel{x^2}\,\cancel{(n+1)}!} \cdot \frac{\cancel{x^2}\, \cancel{n!}}{n\, x^{2n+3}}$$

$$\lim_{n\to\infty}\left|\frac{x^2}{n}\right| < 1 \qquad (-\infty, \infty)$$

$$\sum \frac{(-1)^n x^n}{\sqrt{n+3}}$$

$$\lim_{n\to\infty} \left| \frac{\dfrac{(-1)^{n+1} x^{n+1}}{\sqrt{n+1+3}}}{\dfrac{(-1)^n x^n}{\sqrt{n+3}}} \right| = \lim_{n\to\infty} \left| \frac{-1\cdot x \cdot \sqrt{n+3}}{\sqrt{n+4}} \right|$$

$$= |x| < 1$$

$$-1 < x < 1$$

$$f(x) = \frac{\sin x}{x}$$

(a) Maclaurin series

(b) radius of convergence

(c) 5th order Taylor to estimate $\int_0^1 \frac{\sin x}{x}\,dx$

$$\int_0^1 \frac{\sin x}{x}\,dx = \int_0^1 T_5(x)\,dx$$

$$T(x) = f(a) + \frac{f'(a)\cdot(x-a)}{1} + \frac{f''(a)(x-a)^2}{2!} + \cdots + \cdots \frac{f^{(n)}(a)(x-a)^n}{n!}$$

$$M(x) = f(0) + \frac{f'(0)(x)}{1} + \frac{f''(0)\,x^2}{2!} + \frac{f^{(3)}(0)\,x^3}{3!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$f(x) = \frac{\sin x}{x}$$

$$\frac{x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots}{x} = 1 - \frac{x^2}{3!} + \frac{x^4}{5!} - \ldots,$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n+1)!}$$

we can view the n-th degree Taylor polynomial $T_n(x)$ as a partial sum for the series

$$f(c) + f'(c)\cdot(x-c) + \frac{f''(c)}{2!}(x-c)^2 + \ldots + \frac{f^{(n)}(c)}{n!}(x-c)^n + \ldots$$

This series is called the *Taylor series* for *f* at $x = c$.

The special case of Taylor series for *f*, when $c = 0$, is called the *Maclaurin series*. It is given by

$$f(0) + f'(0)\cdot x + \frac{f''(0)}{2!}x^2 + \frac{f'''(0)}{3!}x^3 + \ldots + \frac{f^{(n)}(0)}{n!}x^n + \ldots = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)}{n!}x^n$$

Series	Converges for:
$\frac{1}{1-x}=1+x+x^2+x^3\ldots+x^n+\ldots=\sum_{n=0}^{\infty}x^n$	$-1<x<1$
$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots+\frac{x^n}{n!}+\ldots=\sum_{n=0}^{\infty}\frac{x^n}{n!}$	all real numbers x
$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\ldots+(-1)^n\frac{x^{2n+1}}{(2n+1)!}+\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$	all real numbers x
$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\ldots+(-1)^n\frac{x^{2n}}{(2n)!}+\ldots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$	all real numbers x

Table 9-2. Some of the important Maclaurin series*