$A = B$

need to show that $A \subseteq B$ and $B \subseteq A$

To get $A \subseteq B$,

Given $x \in A$, to show that

$x \in B$.

1.2

3. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Proof:

$\subseteq$: Let $x \in A \cap (B \cup C)$.

$\Rightarrow x \in A$ and $x \in B \cup C$.

$\Rightarrow x \in A$ and $(x \in B$ or $x \in C)$.

Case 1: $x \in A$ and $x \in B$. $\Rightarrow x \in A \cap B$.

Since $A \cap B \subseteq (A \cap B) \cup (A \cap C)$,

$x \in (A \cap B) \cup (A \cap C)$.

Case 2: $x \in A$ and $x \in C$ $\Rightarrow x \in A \cap C$

Since $A \cap C \subseteq (A \cap B) \cup (A \cap C)$,

$x \in (A \cap B) \cup (A \cap C)$.

$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

$\supseteq$: Let $y \in (A \cap B) \cup (A \cap C)$.

$\Rightarrow y \in (A \cap B)$ or $y \in (A \cap C)$.

Case 1: $y \in (A \cap B) \Rightarrow y \in A$ and $y \in B$.

$y \in B \Rightarrow y \in B \cup C$.

Since $y \in A$ and $y \in B \cup C$, $y \in A \cap (B \cup C)$.

Case 2: $y \in A \cap C \Rightarrow y \in A$ and $y \in C$.

$y \in C \Rightarrow y \in B \cup C$.

Since $y \in A$ and $y \in B \cup C$, $y \in A \cap (B \cup C)$. $\square$

$x \in A \cup B \Rightarrow x \in A$ or $x \in B$

$x \notin A \cup B \Rightarrow x \notin A$ and $x \notin B$

$x \in A \cap B \Rightarrow x \in A$ and $x \in B$

$x \notin A \cap B \Rightarrow x \notin A$ or $x \notin B$

$x \in A \setminus B \Rightarrow x \in A$ and $x \notin B$

$(x \in A - B)$

$x \notin A \setminus B \Rightarrow x \notin A$ or $x \in B$

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

(De Morgan's Laws)

Proof:

$\subseteq$: Let $x \in A \setminus (B \cap C) \Rightarrow x \in A$ and $x \notin B \cap C$

$\Rightarrow x \in A$ and $(x \notin B$ or $x \notin C)$.

Case 1: $x \in A$ and $x \notin B \Rightarrow x \in A - B$

Since $A - B \subseteq (A-B) \cup (A-C)$,

$x \in (A-B) \cup (A-C)$.

Case 2: $x \in A$ and $x \notin C$

(proof similar)

$$A \setminus (B \cap C) = (A \setminus B) \cup (A \setminus C)$$

$\supseteq$: Let $y \in (A-B) \cup (A-C)$

$\Rightarrow y \in A-B$ or $y \in A-C$

case 1: $y \in A-B \Rightarrow y \in A$ and $y \notin B$.

$y \notin B \Rightarrow y \notin B \cap C$

$y \in A$ and $y \notin B \cap C \Rightarrow y \in A-(B \cap C)$.

Case 2: $y \in A-C$

(proof similar)

$\square$

case 1 : $x \geq 0$ and $y \geq 0$

$|xy| = xy \qquad |x||y| = xy$

case 2: $x \geq 0 \quad y < 0$

$|xy| = x(-y) \qquad |x||y| = (x)(-y)$

case 3: $x \leq 0$, $y > 0$

$|xy| = (-x)(y)$ or if $x=0$ $(x)(y)$

$|x||y| = (-x)(y)$

case 4: $x < 0 \quad y < 0$

$|xy| = (-x)(-y) \qquad |x||y| = (-x)(-y)$

0

case 1 : $x \geq 0$ and $y \geq 0$

$|xy| = xy = |x||y|$

case 2: $x \geq 0 \quad y < 0$

$|xy| = x(-y) = |x||y|$

case 3: $x \leq 0$, $y > 0$

$|xy| = (-x)(y) = |x||y|$

case 4: $x < 0 \quad y < 0$

$|xy| = (-x)(-y) = |x||y|$

$$2xy \le x^2 + y^2$$

$$(x-y)^2 \ge 0$$

$$(x-y)^2 = x^2 - 2xy + y^2 \ge 0$$

$$x^2 + y^2 \ge 2xy$$