Chapter 10 - Similarity

Def: The **ratio** of the number a to the number b is the number $\frac{a}{b}$.

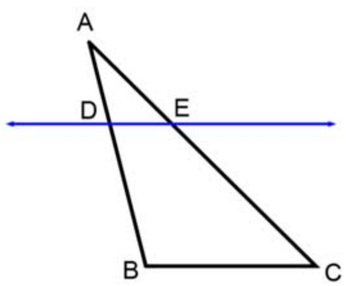
A **proportion** is an equality between ratios. $\frac{a}{b} = \frac{c}{d}$

a, b, c, and d are called the *first, second, third, and fourth terms.*
The second and third terms, b and c, are called the means.
The first and fourth terms, a and d, are called the extremes.

The product of the means is equal to the product of the extremes.
If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

Def: The number b is the **geometric mean** between the numbers a and c if a, b, and c are positive and $\frac{a}{b} = \frac{b}{c}$.

Def: Two triangles are **similar** iff there is a correspondence between their vertices such that their corresponding sides are proportional and their corresponding angles are equal.



Theorem 44 - **The Side-Splitter Theorem**
If a line parallel to one side of a triangle intersects the other two sides in different points,
it divides the sides in the same ratio, that is, if in triangle ABC, DE||BC, then $\frac{AD}{DB} = \frac{AE}{EC}$.

Corollary to the Side-Splitter Theorem:
If a line parallel to one side of a triangle intersects the other two sides in different points, it cuts off segments proportional to the sides, that is, $\frac{AD}{AB} = \frac{AE}{AC}$ and $\frac{DB}{AB} = \frac{EC}{AC}$

Theorem 45 - **The AA Theorem** - If two angles of one triangle are equal to two angles of another triangle, the triangles are similar.

Corollary to the AA Theorem - Two triangles similar to a third triangle are similar to each other.

Theorem 46 - Corresponding altitudes of similar triangles have the same ratio as that of the corresponding sides.

Given: $\Delta ABC \sim \Delta DEF$; BG and EH are altitudes Prove: $\frac{BG}{EH} = \frac{AC}{DF}$

Theorem 47 – The ratio of the perimeters of two similar polygons is equal to the ratio of the corresponding sides.

Given: $\Delta ABC \sim \Delta A'B'C'$ Prove: $\frac{\rho \Delta ABC}{\rho \Delta A'B'C'} = r$, where $r = \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$

Theorem 48 – The ratio of the areas of two similar polygons is equal to the square of the ratio of the corresponding sides.

Given: $\Delta ABC \sim \Delta A'B'C'$ Prove: $\frac{\propto \Delta ABC}{\propto \Delta A'B'C'} = r^2$, where $r = \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CA}{C'A'}$

SAS Similarity Theorem: If an angle of one triangle is equal to an angle of another triangle and the sides including these angles are proportional, then the triangles are similar.

Given: ΔABC and $\Delta A'B'C'$ with $\angle A = \angle A'$ and $\frac{b}{b'} = \frac{c}{c'}$.

Prove: $\Delta ABC \sim \Delta A'B'C'$

SSS Similarity Theorem: If the sides of one triangle are proportional to the sides of another triangle, then the triangles are similar.

Given: ΔABC and $\Delta A'B'C'$ with $\frac{a}{a'} = \frac{b}{b'} = \frac{c}{c'}$.

Prove: $\Delta ABC \sim \Delta A'B'C'$

Chapter 11 – The Right Triangle

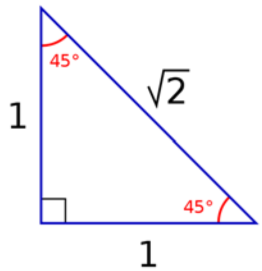
Theorem 49: The altitude to the hypotenuse of a right triangle forms two triangles similar to it and to each other.

Corollary 1 to Theorem 49: The altitude to the hypotenuse of a right triangle is the geometric mean between the segments into which it divides the hypotenuse.

Corollary 2 to Theorem 49: Each leg of a right triangle is the geometric mean between the hypotenuse and its projection on the hypotenuse.

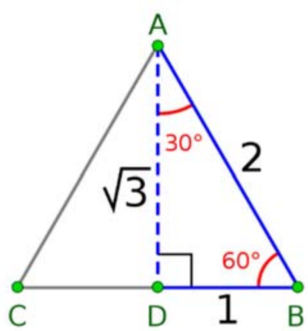
Pythagorean Theorem: In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.

Theorem 50 – The Isosceles Right Triangle Theorem: In an isosceles right triangle, the hypotenuse is $\sqrt{2}$ times the length of a leg.



Corollary to Theorem 50: Each diagonal of a square is $\sqrt{2}$ times the length of one side.

Theorem 51 – The $30° - 60°$ Right Triangle Theorem
In a $30° - 60°$ right triangle, the hypotenuse is twice the shorter leg and the longer leg is $\sqrt{3}$ times the shorter leg.



Corollary to Theorem 51: An altitude of an equilateral triangle having side s is $\frac{\sqrt{3}}{2}s$ and its area is $\frac{\sqrt{3}}{4}s^2$.

Def: The tangent of an acute angle of a right triangle is the ratio of the length of the opposite leg to the length of the adjacent leg.

$$\tan \theta = \text{tangent of } \theta = \frac{\text{opposite}}{\text{adjacent}}$$

Def: The sine of an acute angle of a right triangle is the ratio of the length of the opposite leg to the length of the hypotenuse.

$$\sin \theta = \text{sine of } \theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

Def: The cosine of an acute angle of a right triangle is the ratio of the length of the adjacent leg to the length of the hypotenuse.

$$\cos \theta = \text{cosine of } \theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

Theorem 52: Two nonvertical lines are parallel iff their slopes are equal.

Theorem 53: Two nonvertical lines are perpendicular iff the product of their slopes is -1.

Theorem 54 – The Law of Sines: If the sides opposite $\angle A, \angle B, and\ \angle C$ of ΔABC have lengths $a, b, and\ c$, then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin B}{b}$$

Theorem 55 – The Law of Cosines: If the sides opposite $\angle A, \angle B, and\ \angle C$ of ΔABC have lengths $a, b, and\ c$, then

$$a^2 = b^2 + c^2 - 2bc * \cos A \quad , \quad b^2 = a^2 + c^2 - 2ac * \cos B \quad , \quad c^2 = a^2 + b^2 - 2ab * \cos C$$

Ch 12 – Circles

12.1 - Circles, Radii, and Chords

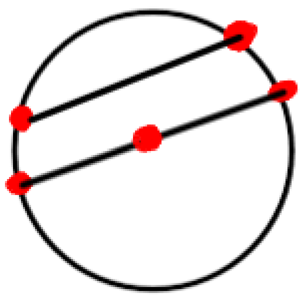
Def: A **circle** is the set of all points in a plane that are at a given distance from a given point in the plane.

Def: Circles are **concentric** iff they lie in the same plane and have the same center.

Def: A **radius** of a circle is a line segment that connects the center of the circle to any point on it.
The radius of a circle is the length of one of these line segments.

Corollary: All radii of a circle are equal.

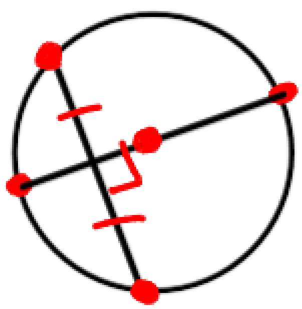
Def: A **chord** of a circle is a line segment that connects two points of the circle.



Def: A **diameter** of a circle is a chord that contains the center.
The diameter of a circle is the length of one of these chords.

Theorem 56: If a line through the center of a circle is perpendicular to a chord,
it also bisects the chord.

Theorem 57: If a line through the center of a circle bisects a chord that is not a diameter, it is also perpendicular to the chord.



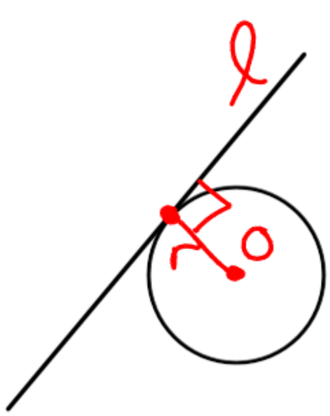
Theorem 58: The perpendicular bisector of a chord of a circle contains the center of the circle.

12.2 - Tangents

Def: A **tangent** to a circle is a line in the plane of the circle that intersects the circle in exactly one point.

Theorem 59: If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact.

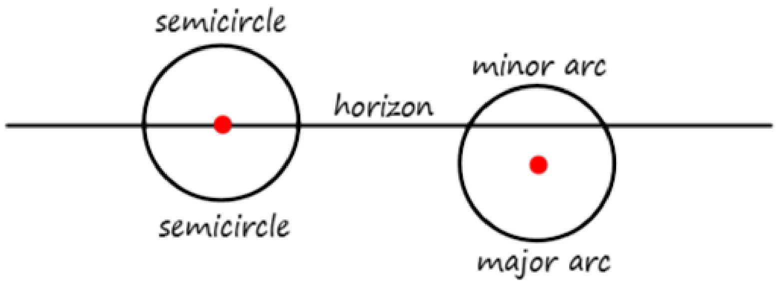
Theorem 60: If a line is perpendicular to a radius at its outer endpoint, it is tangent to the circle.

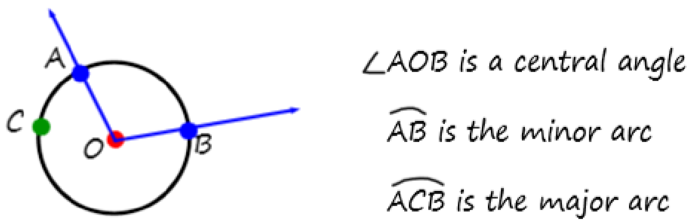


Theorem 59: If a line is tangent to a circle, it is perpendicular to the radius drawn to the point of contact.

Theorem 60: If a line is perpendicular to a radius at its outer endpoint, it is tangent to the circle.

12.3 - Central Angles and Arcs





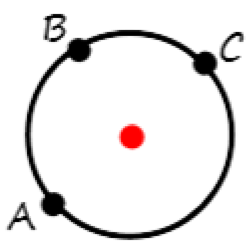
Def: A **central angle** of a circle is an angle whose vertex is the center of the circle.

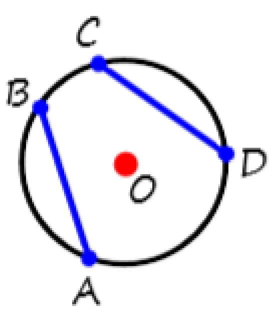
Def: A **reflex angle** is an angle whose measure is more than 180°

Def: The **degree measure** of an arc is the measure of its central angle.

Postulate 10: The Arc Addition Postulate

If C is on $\overset{\frown}{AB}$, then $m\overset{\frown}{AC} + m\overset{\frown}{CB} = m\overset{\frown}{ACB}$



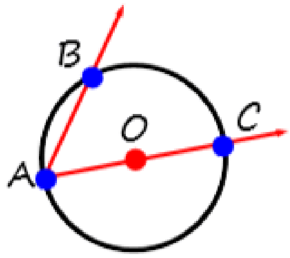


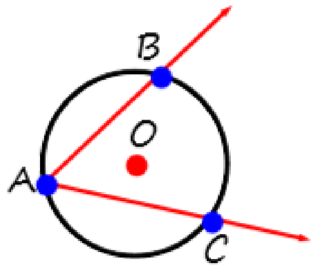
Theorem 61: In a circle, equal chords have equal arcs.

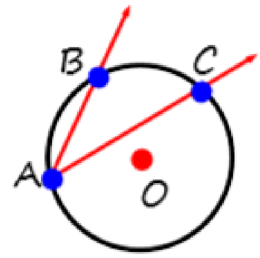
Theorem 62: In a circle, equal arcs have equal chords.

12.4 Inscribed Angles

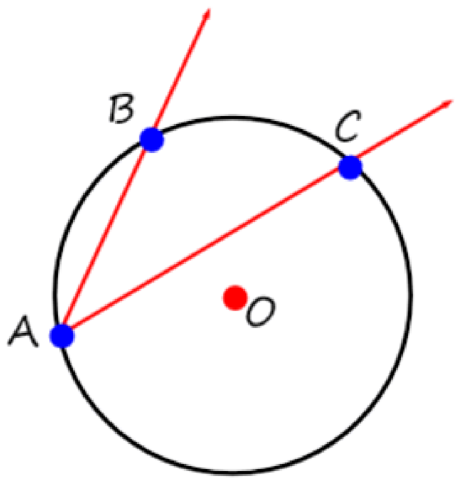
Def: An **inscribed angle** is an angle whose vertex is on a circle, with each of the angle's sides intersecting the circle in another point.



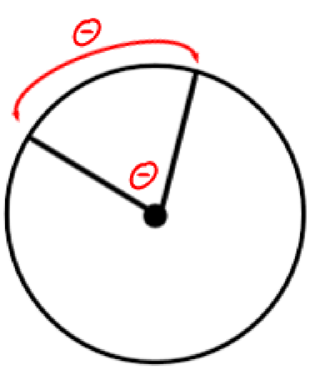


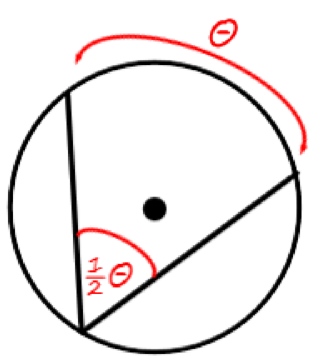


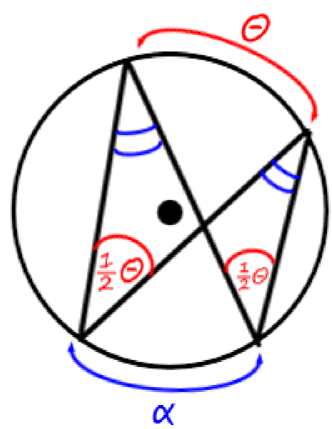
Theorem 63: An inscribed angle is equal in measure to half its intercepted arc.
Corollary 1 to Theorem 63: Inscribed angles that intercept the same arc are equal.
Corollary 2 to Theorem 63: An angle inscribed in a semicircle is a right angle.
Theorem 63: An inscribed angle is equal in measure to half its intercepted arc.

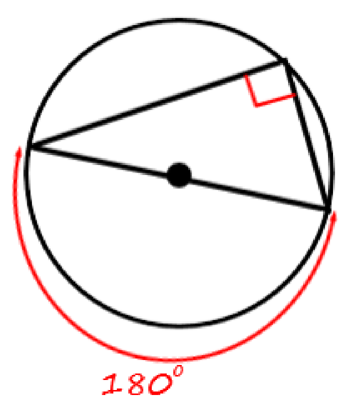


The measure of a central angle is the same as the degree measure of its intercepted arc.
The measure of an inscribed angle is half the degree measure of its intercepted arc.
Inscribed angles with the same intercepted arcs are equal.
Angles inscribed in semicircles are right angles.





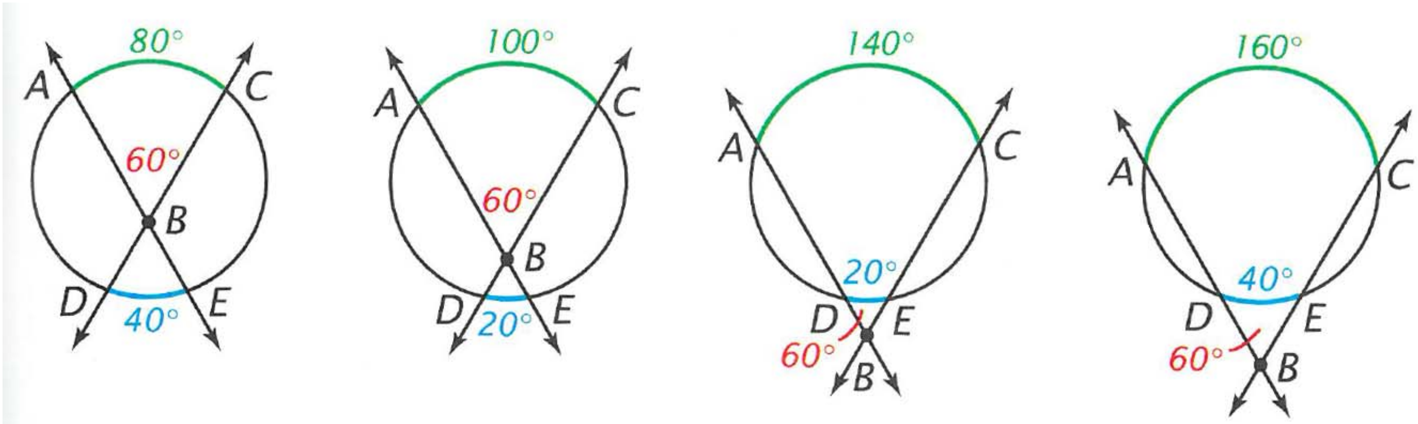




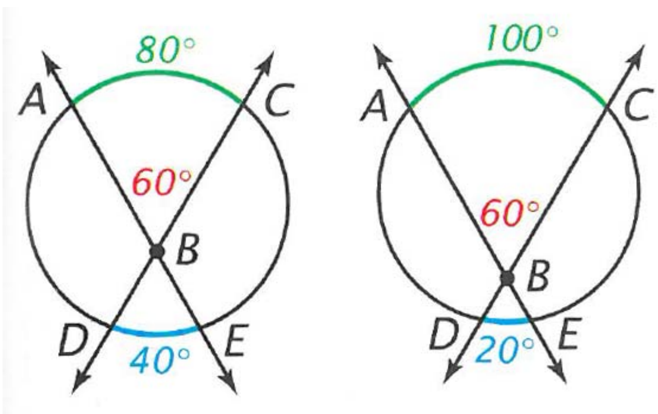
12.5 Secant Angles

Def: A **secant** is a line that intersects a circle in two points.

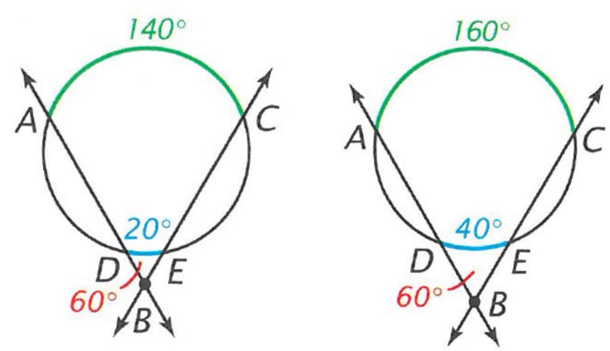
Def: A **secant angle** is an angle whose sides are contained in two secants of a circle so that each side intersects the circle in at least one point other than the angle's vertex.



Theorem 64: A secant angle whose vertex is inside a circle is equal in measure to half the sum of the arcs intercepted by it and its vertical angle.



Theorem 65: A secant angle whose vertex is outside a circle is equal in measure to half the difference of its larger and smaller intercepted arcs.

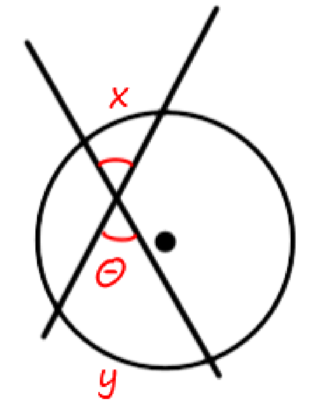


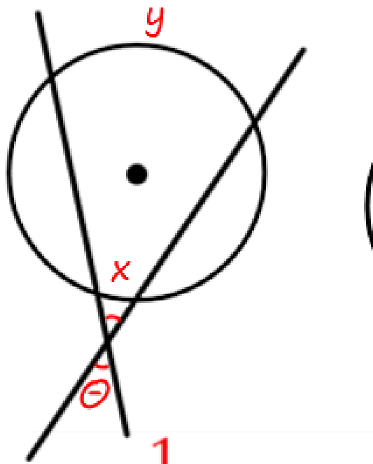
The measure of a secant angle whose vertex is inside the circle is half the sum of its intercepted arcs.

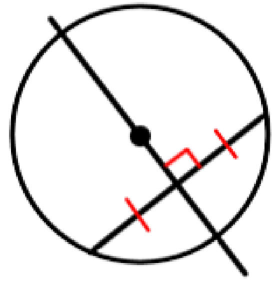
The measure of a secant angle whose vertex is outside the circle is half the difference of its intercepted arcs.

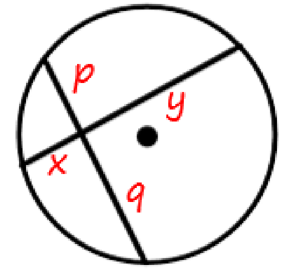
A line through the center of a circle bisects a chord if the two are perpendicular.

If two chords intersect inside a circle, the product of the two segments of one chord is equal to the product of the two segments of the other chord.









$$\theta = \frac{1}{2}(x + y)$$

$$\theta = \frac{1}{2}(y - x)$$

$$xy = pq$$

12.6 - Tangent Segments and Intersecting Chords

Def: If a line is tangent to a circle, then any segment of the line having the point of tangency as one of its endpoints is a **tangent segment** to the circle.

Theorem 66: The tangent segments to a circle from an external point are equal.

Theorem 67: The Intersecting Chords Theorem

If two chords intersect in a circle, the product of the lengths of the segments of one chord is equal to the product of the lengths of the segments of the other chord.

Ch 13 – Concurrence Theorems

Ch 13 - The Concurrence Theorems

13.1 - Triangles and Circles

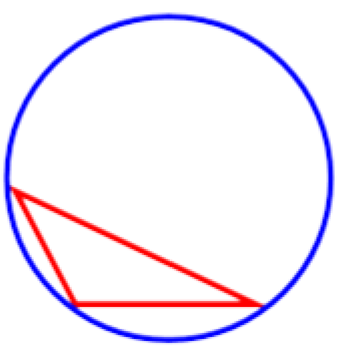
Def: A polygon is **cyclic** iff there exists a circle that contains all of its vertices.

Theorem 68: Every Triangle is cyclic.

Def: A polygon is **inscribed in a circle** iff each vertex of the polygon lies on the circle. The circle is **circumscribed about the polygon**.

Corollary to Theorem 68: The perpendicular bisectors of the sides of a triangle are concurrent.

Construction 9: To circumscribe a circle about a triangle.



13.2 - Cyclic Quadrilaterals

Theorem 69: A quadrilateral is cyclic iff a pair of its opposite angles are supplementary.

13.3 - Incircles

Def: A circle is **inscribed in a polygon** iff each side of the polygon is tangent to the circle. The polygon is **circumscribed about the circle**. The circle is called the **incircle** of the polygon and its center is called the **incenter** of the polygon.

Theorem 70: Every triangle has an incircle.

Corollary to Theorem 70: The angle bisectors of a triangle are concurrent.
Construction 10: To inscribe a circle in a triangle.

13.4 - The Centroid of a Triangle

Def: A **median** of a triangle is a line segment that joins a vertex to the midpoint of the opposite side.
Theorem 71: The medians of a triangle are concurrent.
Def: The **centroid** of a triangle is the point in which its medians are concurrent.
Theorem 72: The lines containing the altitudes of a triangle are concurrent.
Def: The **orthocenter** of a triangle is the point in which the lines containing its altitudes are concurrent.

13.5 - Ceva's Theorem

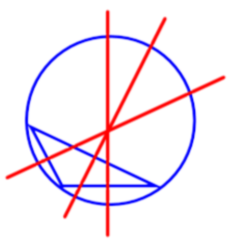
Def: A cevian of a triangle is a line segment that joins a vertex of the triangle to a point on the opposite side.

Theorem 73: Ceva's Theorem

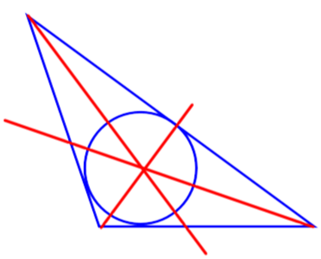
Three cevians, AY, BZ, and CX of ΔABC are concurrent iff

$$\frac{AX}{XB} \cdot \frac{BY}{YC} \cdot \frac{CZ}{ZA} = 1$$

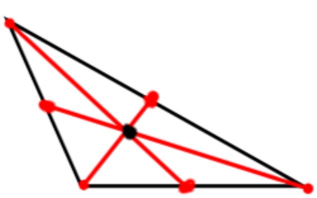
The point at which the perpendicular bisectors of the sides of a triangle are concurrent is the center of a circle circumscribed about the triangle.



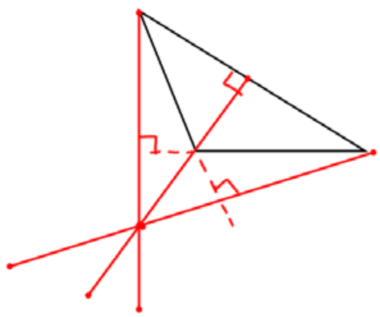
The point at which the angles bisectors of a triangle are concurrent is the center of the incircle, or a circle inscribed within the triangle.



The point at which the medians of a triangle, or line segments joining each vertex to the midpoint of the opposite side, are concurrent is the centroid, or center of mass.



The point at which the lines containing the altitudes of a triangle are concurrent is the <u>orthocenter</u>.



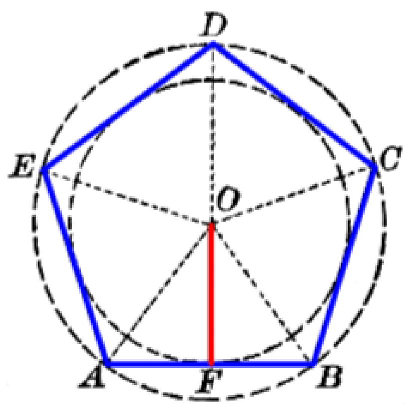
<u>Ch 14 - Regular Polygons and the Circle</u>

<u>14.1 - Regular Polygons</u>

Def: A **<u>regular polygon</u>** is a convex polygon that is both equilateral and equiangular.

<u>Theorem 74</u>: Every regular polygon is cyclic.

Def: An **<u>apothem</u>** of a regular polygon is a perpendicular line segment from its center to one of its sides.



O is the center of pentagon ABCDE

OF is an apothem

OB is a <u>radius</u> of the pentagon (segment connecting the center to a vertex)

BOC is a central angle

<u>14.2 – The Perimeter of a Regular Polygon</u>

Theorem 75 – The perimeter of a regular polygon having n sides is $2Nr$, in which $N = n\sin\frac{180}{n}$ and r is its radius.

Length of one side of a regular n-gon is $2r\sin\frac{180}{n}$

Perimeter of a regular n-gon is $2nr\sin\frac{180}{n}$

14.3 – The Area of a Regular Polygon

Theorem 76 – The area of a regular polygon having n sides is Mr^2, in which $M = n\sin\frac{180}{n}\cos\frac{180}{n}$ and r is its radius.

14.4 – From Polygons to Pi

Def: The circumference of a circle is the limit of the perimeters of the inscribed regular polygons.

Theorem 77 – If the radius of a circle is r, its circumference is $2\pi r$.

Corollary to Theorem 77 – If the diameter of a circle is d, its circumference is πd.

14.5 – The Area of a Circle

Def: The area of a circle is the limit of the areas of the inscribed regular polygons.

Theorem 78 – If the radius of a circle is r, its area is πr^2.

14.6 – Sectors and Arcs

Def: A sector of a circle is a region bounded by an arc of the circle and the two radii to the endpoints of the arc.

If a sector is a certain fraction of a circle, then its area is the same fraction of the circle's area. If an arc is a certain fraction of a circle, then its length is the same fraction of the circle's circumference.

Ch 15 – Geometric Solids

15.1 – Lines and Planes in Space

Postulate 11 – If two points lie in a plane, the line that contains them lies in the plane.

Postulate 12 – If two planes intersect, they intersect in a line.

Def: Two lines are skew iff they are not parallel and do not intersect.

Def: Two planes, or a line and a plane, are parallel iff they do not intersect.

Def: A line and a plane are perpendicular iff they intersect and the line is perpendicular to every line in the plane that passes through the point of intersection.

Def: Two planes are perpendicular iff one plane contains a line that is perpendicular to the other plane.

15.2 – Rectangular Solids

Def: A polyhedron is a solid bounded by parts of intersecting planes.

Def: A rectangular solid is a polyhedron that has six rectangular faces.

Theorem 79 – The length of a diagonal of a rectangular solid with dimensions l, w, and h is $\sqrt{l^2 + w^2 + h^2}$.

Corollary to Theorem 79 – The length of a diagonal of a cube with edges of length e is $e\sqrt{3}$.

15.3 – Prisms

15.4 – The Volume of a Prism

Geometry - Review of Ch 12, 14, & 15

$$\theta = \frac{1}{2}(x + y)$$

$$\theta = \frac{1}{2}(y - x)$$

$$xy = pq$$

$$A = \frac{1}{2}xy$$

$$V = \frac{1}{2}xyh$$

Θ

A

$area\ of\ circular\ base = \pi r^2$

$V_{cylinder} = \pi r^2 h$

$circumference\ of\ circular\ base$
$= 2\pi r$

$lateral\ surface\ area\ of\ cylinder$
$= 2\pi rh$

$total\ surface\ area\ of\ cylinder$
$= 2\pi rh + 2\pi r^2$

$V_{sphere} = \frac{4}{3}\pi r^3$

$surface\ area\ of\ sphere$
$= 4\pi r^2$

$V_{cone} = \frac{1}{3}A_b h$

$A_{base} = \pi r^2$

$V_{cone} = \frac{1}{3}\pi r^2 h$

$lateral\ area\ of\ a\ cone$
$= \pi rl$

$total\ surface\ area\ of\ a\ cone$
$= \pi rl + \pi r^2$

$V_{pyramid} = \frac{1}{3}A_b h$

$lateral\ area\ of\ a\ pyramid$
$= \frac{1}{2}P_b l$

$total\ surface\ area\ of\ a\ pyramid$
$= \frac{1}{2}P_b l + A_b$

$\cos\alpha = \frac{y}{r}$ $\quad\quad$ $y = r\cos\alpha$

$\sin\alpha = \frac{x}{r}$ $\quad\quad$ $x = r\sin\alpha$

We have n triangles whose base is 2x and height is y.

$Area\ of\ polygon = n(r\cos\alpha)(r\sin\alpha)$

$$A = nr^2 \cos\frac{180}{n}\sin\frac{180}{n}$$

$Perimeter\ of\ polygon = n(2x)$

$$P = 2nr\sin\frac{180}{n}$$

$$\theta = \frac{360}{n} \qquad \alpha = \frac{1}{2}\theta \qquad \alpha = \frac{180}{n}$$

Ch 16

Statement	**Euclid**	**Lobachevsky**	**Riemann**
Through a point not on a line, there is	*exactly one* line parallel to the line.	*more than one* line parallel to the line.	*no line* parallel to the line.
The summit angles of a Saccheri quadrilateral are	right.	acute.	obtuse.