Precalculus Graphing Summary

Basic functions to know: $y = x$, $y = |x|$, $y = x^2$, $y = \sqrt{x}$, $y = x^3$, $y = \sqrt[3]{x}$

Graphing $\boldsymbol{y = af(bx + c) + d}$ from $\boldsymbol{y = f(x)}$:

$y = af(x)$

A constant a multiplied outside a function results in a vertical shrink or stretch. Graph $y = f(x)$ first. If $|a| > 1$, stretch the graph vertically by a factor of a. If $|a| < 1$, shrink the graph vertically by a factor of a. For a given x-value in one of the basic graphs, multiply the y-value by a to find the new y-value for the transformed graph. If $a < 0$, flip the graph vertically.

$y = f(bx)$

A constant b multiplied inside a function results in a horizontal shrink or stretch. Graph $y = f(x)$ first. If $|b| > 1$, shrink the graph horizontally by a factor of b. If $|b| < 1$, stretch the graph horizontally by a factor of b. Note that this is the opposite of what happens for vertical stretching/shrinking. For a given y-value in one of the basic graphs, multiply the x-value by b to find the new x-value for the transformed graph. If $b < 0$, flip the graph horizontally.

$y = f(x) + d$

A constant added outside a function results in a vertical shift. If $d > 0$, shift up by d units. If $d < 0$, shift down by d units.

$y = f(x + c)$

A constant added inside a function results in a horizontal shift. If $c > 0$, shift left by c units. If $c < 0$, shift right by c units.

Graphing $\boldsymbol{y = f^{-1}(x)}$ from $\boldsymbol{y = f(x)}$:

Reflect the graph of $f(x)$ over the line y=x by interchanging all x- and y-values.

x-intercepts are the points (x,0) at which the graph crosses the x-axis. The x-values themselves are called zeros. It is also important to note the multiplicity of your zeros--this is the number of times the factor that yields the zero occurs or is multiplied by itself (the exponent on the factor). Odd multiplicity means the graph crosses the x-axis at the zero; even multiplicity means the graph bounces off the x-axis at the zero.

To find: set $f(x) = 0$ and solve for x. For polynomials that are factored, set each factor = 0 and solve for x. For polynomials that are hard to factor, find possible rational zeros by looking at factors of the constant term over factors of the leading coefficient, and perform long division to find factors. For rational functions and other functions with fractions, it is enough to set the denominator = 0 and solve for x.*

vertical asymptotes are vertical lines which the graph of a function approaches but never touches. They occur at x-values for which the function is undefined. Always have equation of the form $x = c$.

To find: set the denominator = 0 and solve for x.*

*except when the same values make both the numerator and the denominator = 0, in which case there is not an asymptote, but instead a hole in the graph at that x-value

y-intercepts are the points (0,y) at which the graph crosses the y-axis. The graph of a function should only have one of these if any.
To find: plug 0 in for x.

lead term test for polynomials tells us the end behavior of the graph. Even polynomials (the degree or largest exponent of the polynomial is even) behave like $y = x^2$ -- positive y-values as x approaches both $\pm\infty$; odd polynomials behave like $y = x^3$-- positive y-values as x approaches$+\infty$, negative y-values as x approaches $-\infty$. A negative leading coefficient will yield these results upside down.

horizontal asymptotes are horizontal lines ($y = c$) that determine the end behavior of the graph a function. As x goes to $\pm\infty$ (the far right and left sides of your graph), the graph will approach these lines, may cross them, and may even lie exactly on them.
To find: look at the ratio of lead terms. If the degree of the numerator and denominator is the same, the H.A. is the ratio of leading coefficients. If the degree of the numerator is smaller than the degree of the denominator, the H.A. is $y = 0$. If there are radicals or fractions present, take those into account following order of operations.

oblique/slant asymptotes are diagonal lines (y=mx+b) that determine the end behavior of the graph of a function. A graph will have either horizontal or oblique asymptotes, but never both.
To find: these occur when the degree of the numerator is 1 higher than the degree of the denominator. Found using long division (numerator divided by denominator)

Fractional exponents ($y = x^{m/n}$)
even denominator: have no symmetry since the domain consists of only positive values; think of $y = \sqrt{x} = x^{1/2}$ as a reference; for fractions larger than 1, the graph becomes concave up
odd numerator, odd denominator: have origin symmetry; think of $y = \sqrt[3]{x} = x^{1/3}$ as reference
even numerator, odd denominator: have y-axis symmetry; the even numerator means that this is just the odd numerator, odd denominator type that has been raised to an even power, and raising things to an even power makes them positive; graph the odd,odd function first, then flip the negative half over the x-axis

Negative fractional exponents
Recall that negative exponents just mean to take the reciprocal. You can get the graph of $y = x^{-n}$ from $y = x^n$ (whether n is an integer or a fraction), by examining domain, vertical asymptotes, zeros, and end behavior. The values that made $x^n = 0$ are the same values that make $y = x^{-n}$ undefined, so where the former has zeros, the latter has vertical asymptotes