Topology HW #1 – Munkres Section 1.1 – Set Theory and Logic

The **union** of two sets A and B is $A \cup B = \{x | x \in A \text{ or } x \in B\}$.
The **intersection** of two sets A and B is $A \cap B = \{x | x \in A \text{ and } x \in B\}$.
The **difference** of two sets A and B is $A - B = \{x | x \in A \text{ and } x \notin B\}$.
An **arbitrary union** is $\bigcup_{A \in \mathcal{A}} A = \{x | x \in A \text{ for at least one } A \in \mathcal{A}\}$.
An **arbitrary intersection** is $\bigcap_{A \in \mathcal{A}} A = \{x | x \in A \text{ for every } A \in \mathcal{A}\}$.
The **power set** of A, denoted by $\mathcal{P}(A)$ is the set of all subsets of A.
The **cartesian product** of sets A and B is $A \times B = \{(x, y) | x \in A \text{ and } y \in B\}$.
For a statement "If A, then B," the **converse** is "If B, then A" and the **contrapositive** is "If not B, then not A."
A statement "There exists an element $a \in A$ such that P is true" has **negation** "For all elements $a \in A$, P is not true."
A statement "For all elements $a \in A$, P is true" has **negation** "There exists an element $a \in A$ such that P is not true."
$x \notin A \cup B \Rightarrow x \notin A \text{ and } x \notin B$; $x \notin A \cap B \Rightarrow x \notin A \text{ or } x \notin B$
$(x, y) \notin A \times B \Rightarrow x \notin A \text{ or } y \notin B$; $x \notin A - B \Rightarrow x \notin A \text{ or } x \in B$
To show that a statement is true, prove it in general; to show that a statement is false, provide a specific counter-example.
To prove that two sets are equal ($A = B$), prove both $A \subseteq B$ and $B \subseteq A$.
To show $A \subseteq B$, choose an arbitrary element $x \in A$ and build an argument that leads to $x \in B$.

Problems NOT worked in class that were left for homework:

1. Distributive laws & DeMorgan's laws – need to prove both $\subseteq$ and $\supseteq$.
$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
$A - (B \cap C) = (A - B) \cup (A - C)$

2. c, d Determine which of $\Leftrightarrow, \Rightarrow, or \ \Leftarrow$ is true by proving or providing counter-example(s)
e, f, g, h, n, q Determine which of $=, \subseteq, or \ \supseteq$ is true by proving or providing counter-example(s)

3. Determine if original statement is true or false.
Write the contrapositive of the statement and determine if it is true or false.
Write the converse of the statement and determine if it is true or false.

4. Write the negation of each statement.

5. Determine if the original statement is true or false.
Write the converse of the statement and determine if it is true or false.

6. Write the contrapositive of each statement in #5.

7. Express sets D and E in terms of sets A, B, and C using the symbols $\cup, \cap, and\ -$.

8. Construct the power sets and determine the number of elements for each of the sets. (We said in class that it's called the "power set" because the number of elements in the power set is two raised to the power of the number of elements in the original set.)
$A_2 = \{a, b\}$; $A_1 = \{a\}$; $A_3 = \{a, b, c\}$; $A_0 = \emptyset$

9. $A - \bigcup_{B \in \mathfrak{B}} B = \bigcap_{B \in \mathfrak{B}} (A - B)$ (need to prove both $\subseteq$ and $\supseteq$)
$A - \bigcap_{B \in \mathfrak{B}} B = \bigcup_{B \in \mathfrak{B}} (A - B)$ (need to prove $\supseteq$)

10. Find the two subsets of $\mathbb{R}$ that each set is equivalent to the Cartesian product of OR provide a counter-example.