Special Factoring

$a^2 - b^2 = (a-b)(a+b)$

$(a+b)^2 = a^2 + 2ab + b^2$

$(a-b)^2 = a^2 - 2ab + b^2$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Converting Between Degree & Radian Measure

To convert from degree to radian measure, multiply by $\frac{\pi}{180°}$

To convert from radian to degree measure, multiply by $\frac{180°}{\pi}$

Trig Functions of an Acute Angle

$\sin\theta = \frac{side\ opposite\ \theta}{hypotenuse}$ $\quad$ $\csc\theta = \frac{hypotenuse}{side\ opposite\ \theta}$

$\cos\theta = \frac{side\ adjacent\ to\ \theta}{hypotenuse}$ $\quad$ $\sec\theta = \frac{hypotenuse}{side\ adjacent\ to\ \theta}$

$\tan\theta = \frac{side\ opposite\ \theta}{side\ adjacent\ to\ \theta}$ $\quad$ $\cot\theta = \frac{side\ adjacent\ to\ \theta}{side\ opposite\ \theta}$

Arc Length and Angular Speed

$s = r\theta\,, \quad v = \frac{s}{t}\,, \quad \omega = \frac{\theta}{t}\,, \quad v = r\omega$

Dimensional analysis conversion factors

$\frac{5280\,ft}{1\,mi}, \frac{12\,in}{1\,ft}, \frac{2\pi}{1\,rev}, \frac{\pi}{180°}, \frac{60\,min}{1\,hr}, \frac{60\,sec}{1\,min}$, & reciprocals

Variables

s = distance traveled or arc length (in, km, mi)

t = time (sec, min, hr, days)

θ = amount of rotation or included angle (deg, rad, rot, rev)

r = radius or distance from the center of rotation (cm, in, ft)

$\mathrm{v} = \text{linear speed} = \frac{\text{distance}}{\text{time}}$

$\omega = \text{angular speed} = \frac{\text{amount of rotation}}{\text{time}}$

Trigonometric Identities

Reciprocal Identities

$\csc x = \frac{1}{\sin x}\ ,\quad \sin x = \frac{1}{\csc x}$

$\sec x = \frac{1}{\cos x}\ ,\quad \cos x = \frac{1}{\sec x}$

$\cot x = \frac{1}{\tan x}\ ,\quad \tan x = \frac{1}{\cot x}$

Ratio Identities

$\tan \mathrm{x} = \frac{\mathrm{sinx}}{\cos \mathrm{x}}\ ,\quad \cot x = \frac{\cos x}{\sin x}$

Odd-Even Identities

$\cos(-x) = \cos x\ ,\ \sin(-x) = -\sin x\ ,\ \tan(-x) = -\tan x$

$\sec(-x) = \sec x\ ,\ \csc(-x) = -\csc x\ ,\ \cot(-x) = -\cot x$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \cot^2 x = \csc^2 x$

$\tan^2 x + 1 = \sec^2 x$

Sum and Difference Identities

$\sin(a+b) = \sin a\cos b + \cos a\sin b$

$\sin(a-b) = \sin a\cos b - \cos a\sin b$

$\cos(a+b) = \cos a\cos b - \sin a\sin b$

$\cos(a-b) = \cos a\cos b + \sin a\sin b$

$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a\tan b}$

$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a\tan b}$

Cofunction Identities

$\sin\left(\frac{\pi}{2} - x\right) = \cos x\ ,\quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$

$\tan\left(\frac{\pi}{2} - x\right) = \cot x\ ,\quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$

$\csc\left(\frac{\pi}{2} - x\right) = \sec x\ ,\quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$

Double-Angle Identities

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x = 2\cos^2 x - 1 = 1 - 2\sin^2 x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

Half-Angle Identities

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}\ ,\quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{\sin x}{1+\cos x} = \frac{1-\cos x}{\sin x}$

Solving Triangles

Law of Sines

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of Cosines

$c^2 = a^2 + b^2 - 2ab\cos C$

$b^2 = a^2 + c^2 - 2ac\cos B$

$a^2 = b^2 + c^2 - 2bc\cos A$

Area of a Triangle

$K = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B = \frac{1}{2}ab\sin C$

Vectors

The **component form** of $\overrightarrow{AC}$ with $A = (x_1, y_1)$ and $C = (x_2, y_2)$ is $\overrightarrow{AC} = \langle x_2 - x_1,\ y_2 - y_1 \rangle$

The **magnitude** of a vector $\vec{v}$ with component form $\langle a, b \rangle$ is $|\vec{\boldsymbol{v}}| = \sqrt{\boldsymbol{a^2 + b^2}}$

The reference angle for the **direction angle** θ of the vector $\langle a, b \rangle$ is given by $\theta = \tan^{-1}\frac{b}{a}$. Figure out which quadrant this angle should be in and measure the angle counterclockwise from the positive x-axis.

The **horizontal component** of the vector $\langle a, b \rangle$ is $a = |\vec{v}| \cos\theta$

The **vertical component** of the vector $\langle a, b \rangle$ is $b = |\vec{v}| \sin\theta$

For a real number k and a vector $\vec{v} = \langle v_1, v_2 \rangle$, the **scalar product** of k and $\vec{v}$ is $k\vec{v} = k\langle v_1, v_2 \rangle = \langle kv_1,\ kv_2 \rangle$. The vector $k\vec{v}$ is a **scalar multiple** of the vector $\vec{v}$.

Vector Addition/Subtraction: If $\vec{u} = \langle u_1,\ u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$, then $\vec{\boldsymbol{u}} \pm \vec{\boldsymbol{v}} = \langle \boldsymbol{u_1} \pm \boldsymbol{v_1},\ \boldsymbol{u_2} \pm \boldsymbol{v_2} \rangle$.

If $\vec{v}$ is a vector and $\vec{v} \neq \vec{0}$, then $\frac{\vec{\boldsymbol{v}}}{|\vec{\boldsymbol{v}}|}$ is a **unit vector** (vector with magnitude 1) in the direction of $\vec{v}$.

The **dot product** of two vectors $\vec{u} = \langle u_1,\ u_2 \rangle$ and $\vec{v} = \langle v_1, v_2 \rangle$ is $\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}} = \boldsymbol{u_1 v_1 + u_2 v_2}$.

If θ is the **angle between two nonzero vectors** $\vec{u}$ and $\vec{v}$, then $\boldsymbol{\cos\theta} = \frac{\vec{\boldsymbol{u}} \cdot \vec{\boldsymbol{v}}}{|\vec{\boldsymbol{u}}||\vec{\boldsymbol{v}}|}$.

Trigonometric Form of Complex Numbers

A **complex number** $z = a + bi$, where $i = \sqrt{-1}$ can be written in **trigonometric form** as $z = r(\cos\theta + i\sin\theta)$ or $z = r \operatorname{cis}\theta$, where $r = \sqrt{a^2 + b^2}$ is the **modulus** of z and direction angle θ is referred to as the **argument**.

$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)] = r_1 r_2 \operatorname{cis}\theta$$
$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)] = \frac{r_1}{r_2} \operatorname{cis}\theta$$
$$z^n = r^n[\cos(n\theta) + i\sin(n\theta)] = r^n \operatorname{cis}(n\theta)$$