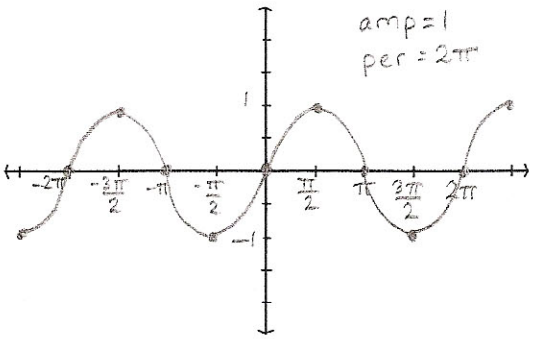
TRIGONOMETRY – GUIDE TO GRAPHING

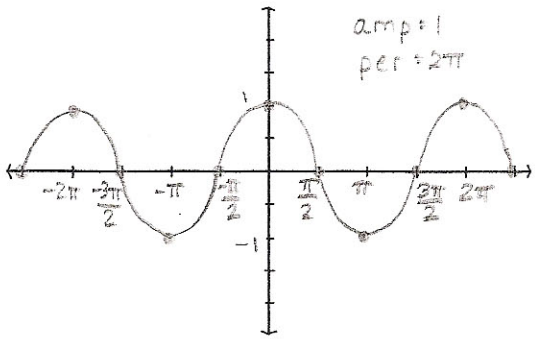
Sarah Gelsinger Brewer - Alabama School of Math and Science 2009

Six basic trigonometric functions:

$y = \sin x$

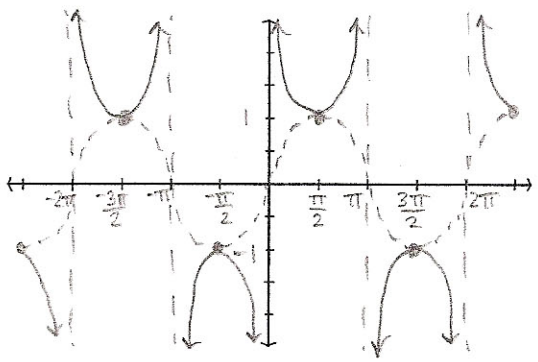


$y = \cos x$



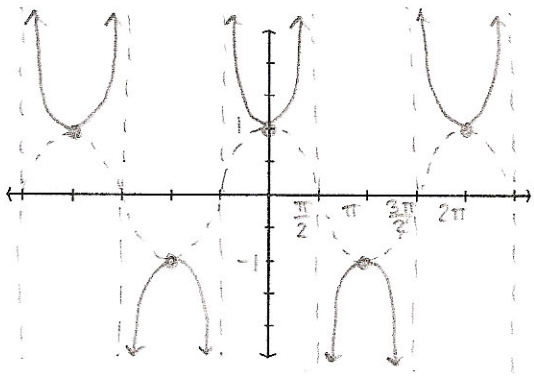
$y = \csc x = \frac{1}{\sin x}$

no zeros; asymptotes when $\sin x = 0$



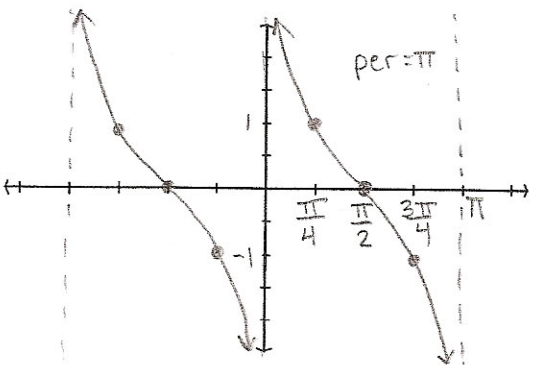
$y = \sec x = \frac{1}{\cos x}$

no zeros; asymptotes when $\cos x = 0$



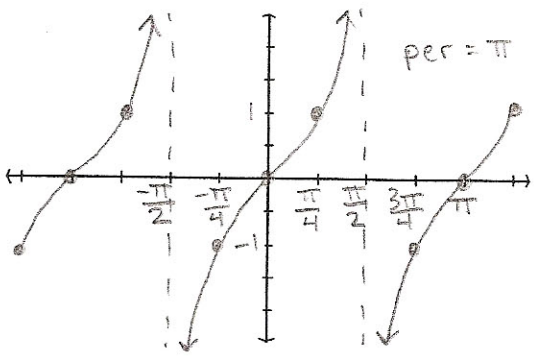
$y = \cot x = \frac{\cos x}{\sin x}$

zeros when $\cos x = 0$; asymptotes when $\sin x = 0$



$y = \tan x = \frac{\sin x}{\cos x}$

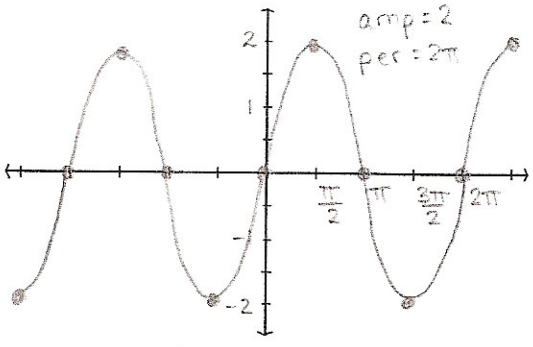
zeros when $\sin x = 0$; asymptotes when $\cos x = 0$



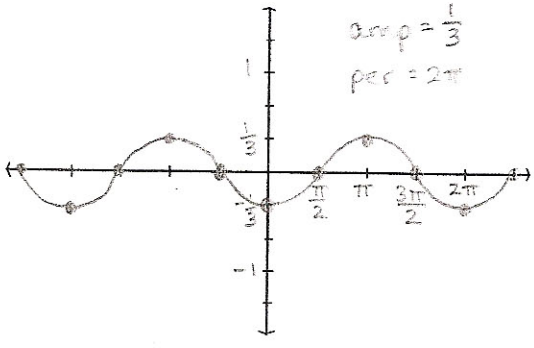
$y = af(x)$

For $y = \sin x$ and $y = \cos x$, $|a|$ is the amplitude of the function. If $a < 0$, flip the graph vertically.

Ex. $y = 2\sin x$

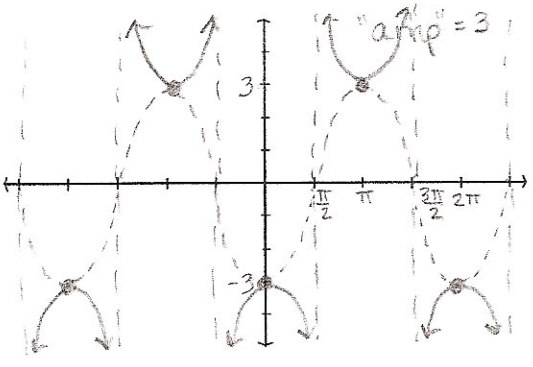


$y = -\frac{1}{3}\cos x$

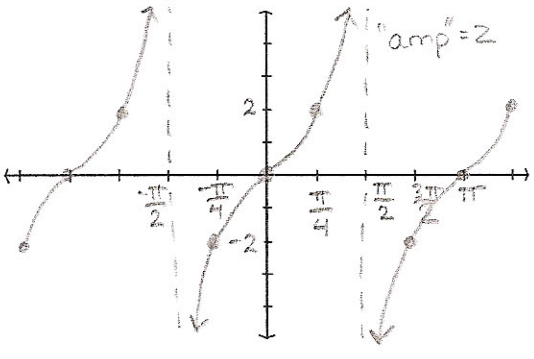


For $y = \sec x$, $y = \csc x$, $y = \tan x$, and $y = \cot x$, $|a|$ is the "amplitude" of reference points of the function. For $\sec x$ and $\csc x$, these are the maximum and minimum points, and for $\tan x$ and $\cot x$, these are the points halfway in between the zeros and vertical asymptotes.

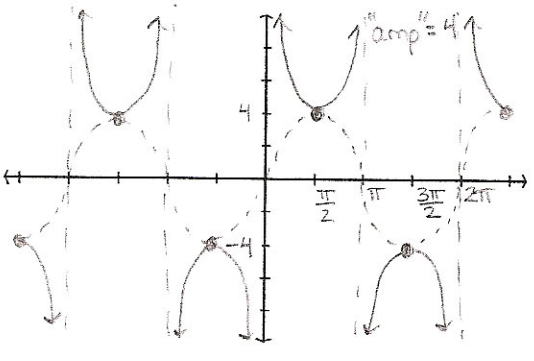
Ex. $y = -3\sec x$



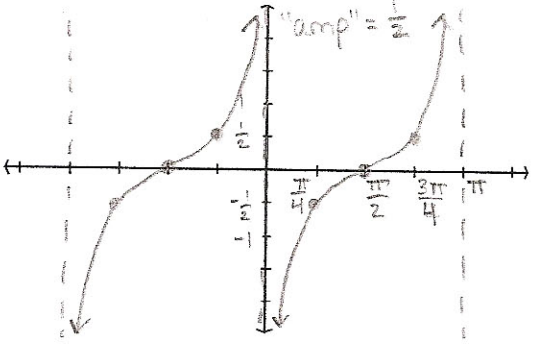
$y = 2\tan x$



Ex. $y = 4\csc x$



$y = -\frac{1}{2}\cot x$



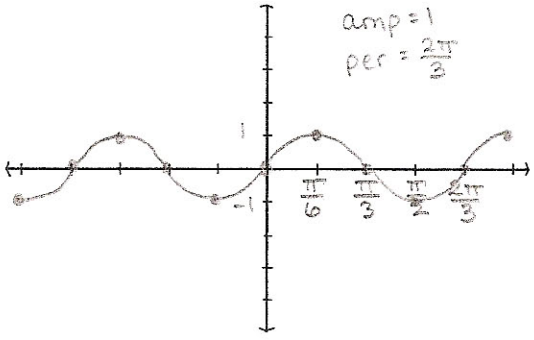
$y = f(bx)$

The period of $y = \sin x, y = \cos x, y = \sec x$, and $y = \csc x$ is 2π. The period of $y = \tan x$ and $y = \cot x$ is π. This is how often the graph repeats itself. When x is multiplied by a constant, the period changes.

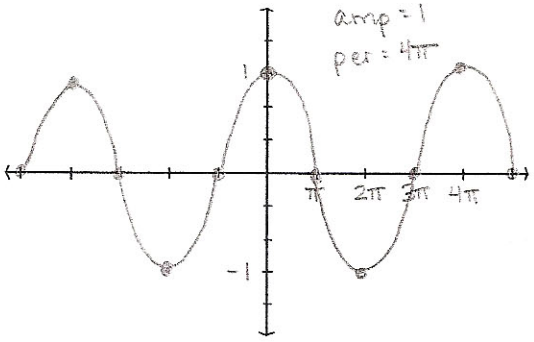
$$new\ period = \frac{period\ of\ original\ function\ (\pi\ or\ 2\pi)}{|b|}$$

To graph, label the 4th tick mark with the new period, divide by 2 to get the 2nd tick mark label, divide the 2nd by 2 to get the 1st tick mark label, and multiply the 1st by 3 to get the 3rd tick mark label. Zeros and asymptotes will correspond to same tick marks as in original graph, but the labels will be different.

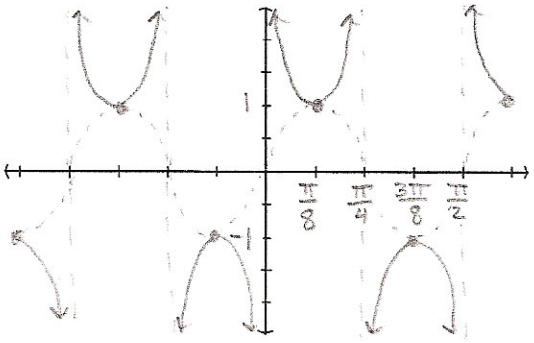
Ex. $y = \sin 3x$, period = $\frac{2\pi}{3}$



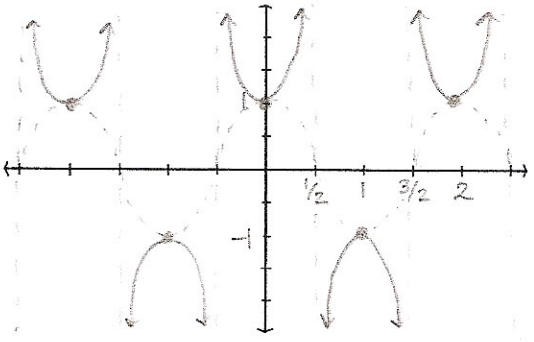
$y = \cos\frac{1}{2}x$, period = $\frac{2\pi}{1/2} = 2\pi \cdot \frac{2}{1} = 4\pi$



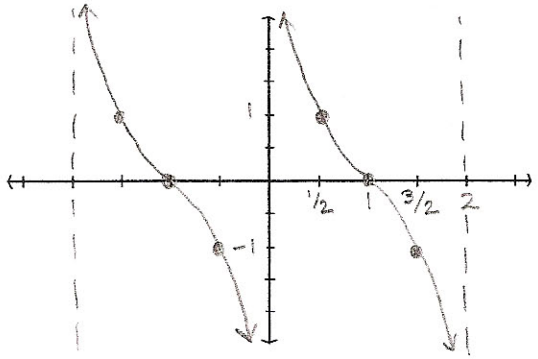
Ex. $y = \csc 4x$, period = $\frac{2\pi}{4} = \frac{\pi}{2}$



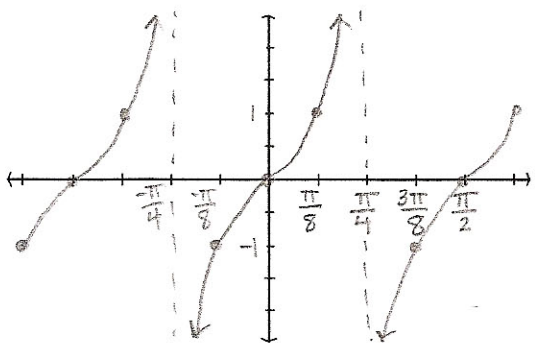
$y = \sec \pi x$, period = $\frac{2\pi}{\pi} = 2$



Ex. $y = \cot\frac{\pi}{2}x$, period = $\frac{\pi}{\pi/2} = \pi \cdot \frac{2}{\pi} = 2$



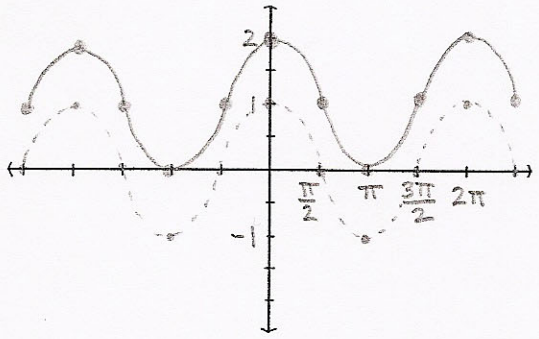
$y = \tan 2x$, period = $\frac{\pi}{2}$



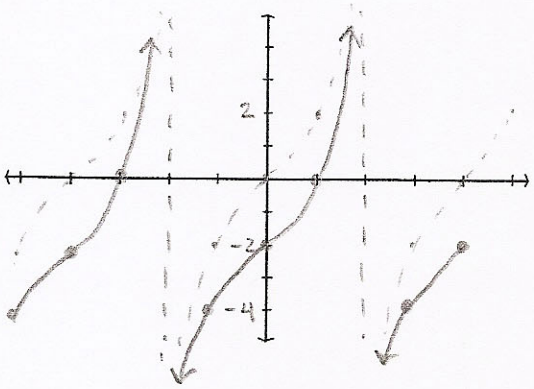
$y = f(x) + d$

d is a <u>vertical shift</u>. If d>0, shift up, if d<0, shift down. First, draw your basic graph with a dotted line, then shift each of your reference points by d and fill in the graph.

Ex. $y = \cos x + 1$ up 1



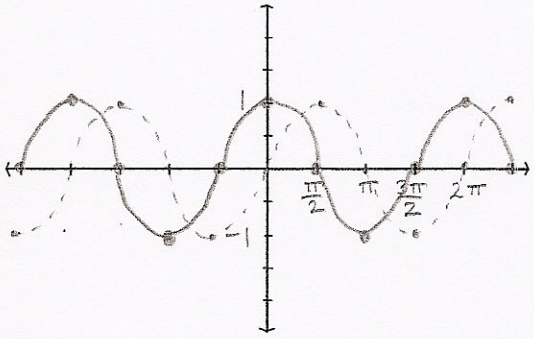
$y = \tan x - 2$ down 2



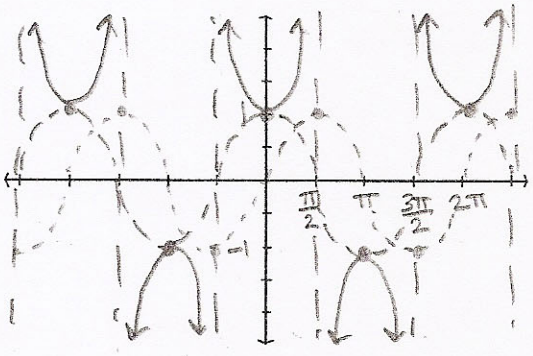
$y = f(x + c)$

c is a <u>horizontal shift</u> or <u>phase shift</u>. If c>0, shift left, if c<0, shift right. First, draw your basic graph with a dotted line, then shift each of your reference points by c and fill in the graph. Be sure to also shift asymptotes!

Ex. $y = \sin\left(x - \frac{3\pi}{2}\right)$ right $\frac{3\pi}{2}$



$y = \csc\left(x + \frac{\pi}{2}\right)$ left $\frac{\pi}{2}$

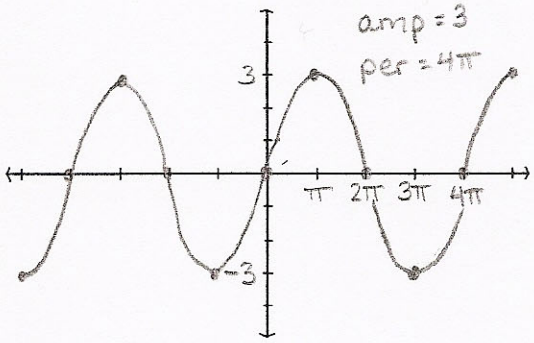


$y = af(bx + c) + d$

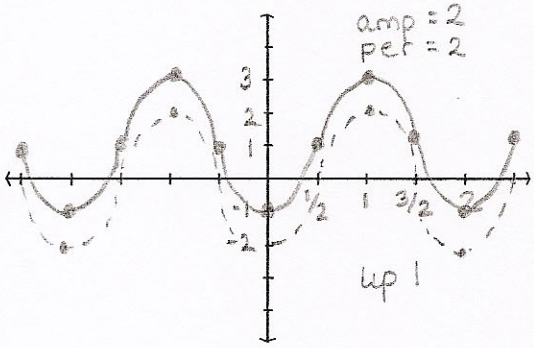
Graph $y = af(bx)$ first with a dotted line, and then shift.

**Remember, when $b \neq 1$ (i.e. any time there IS a b), you must factor b out before horizontal shift!
$y = af(bx + c) + d = af\left[b\left(x + \frac{c}{b}\right)\right] + d$, so the phase shift is $-\frac{c}{b}$.

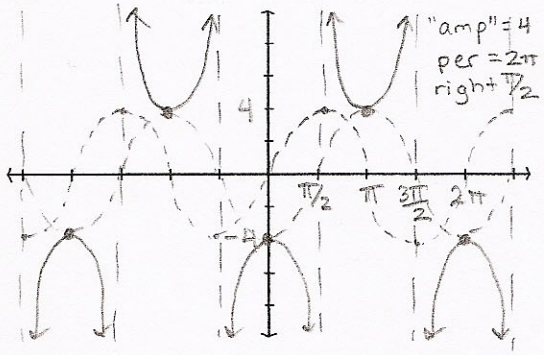
$y = 3\sin\frac{1}{2}x$



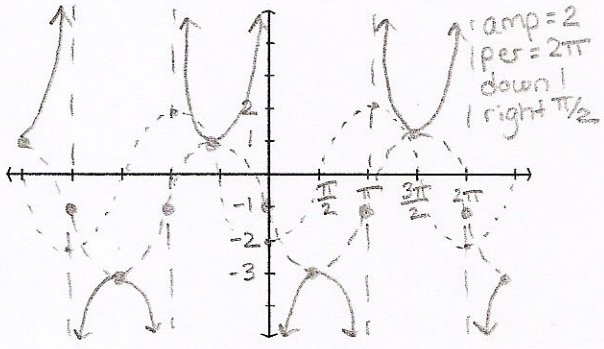
$y = -2\cos\pi x + 1$



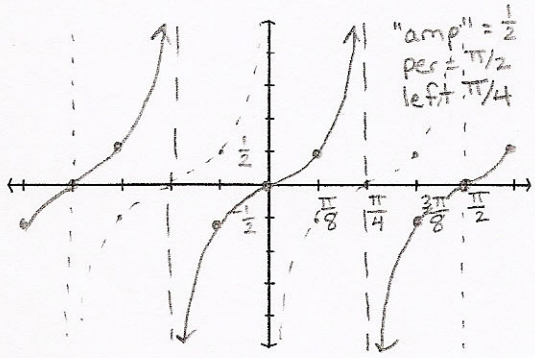
$y = 4\csc\left(x - \frac{\pi}{2}\right)$



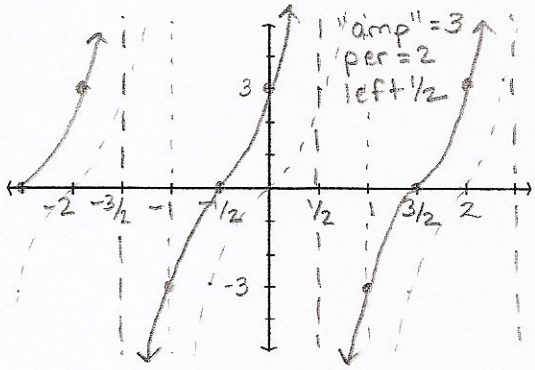
$y = -2\sec\left(x - \frac{\pi}{2}\right) - 1$



$y = -\frac{1}{2}\cot\left(2x + \frac{\pi}{2}\right)$



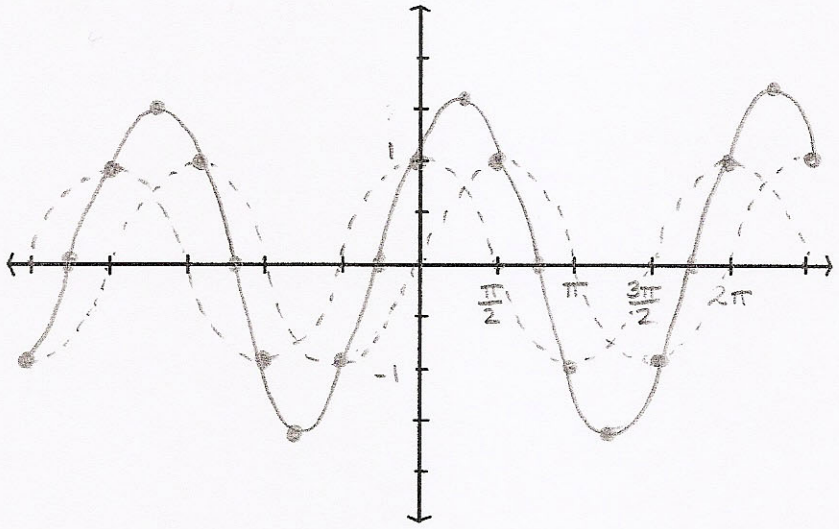
$y = 3\tan\left(\frac{\pi}{2}x + \frac{\pi}{4}\right)$



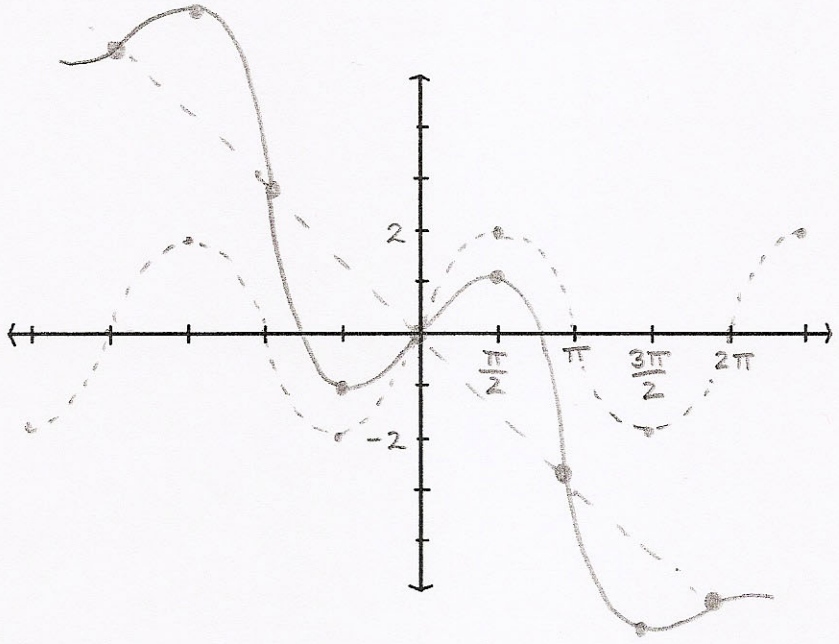
$y = f(x) + g(x)$

Graph $f(x)$ and $g(x)$ on the same graph with dotted lines. If the equation is $y = f(x) - g(x)$, graph $f(x)$ and $-g(x)$ on the same graph with dotted lines. For each x-value, add the y-values of the two functions. The sum is the new function value. In general, choose x-values where both of the graphs are equal, where one graph is equal to zero, where one or both graphs has a maximum or minimum, and where both graphs are approximately the same distance from zero on opposite sides of the x-axis.

$y = \sin x + \cos x$

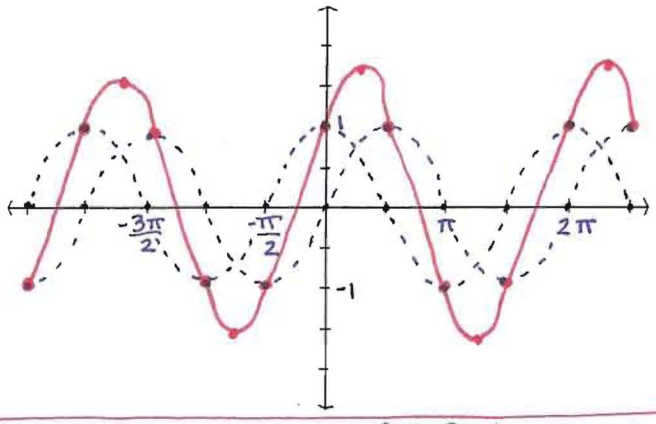


$y = 2\sin x - x$



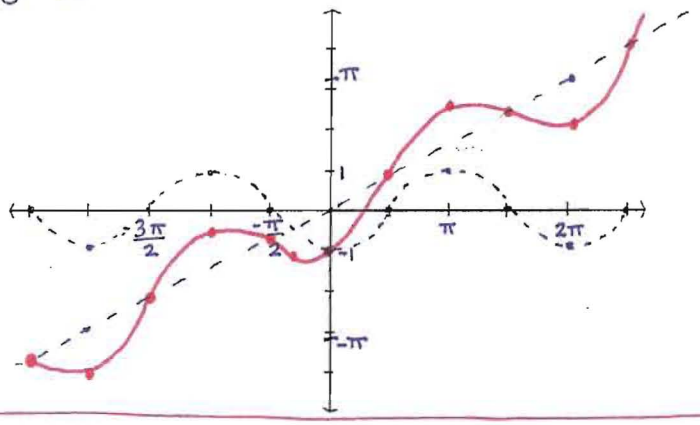
Yesterday in class we graphed:

$y = \sin x + \cos x$



&

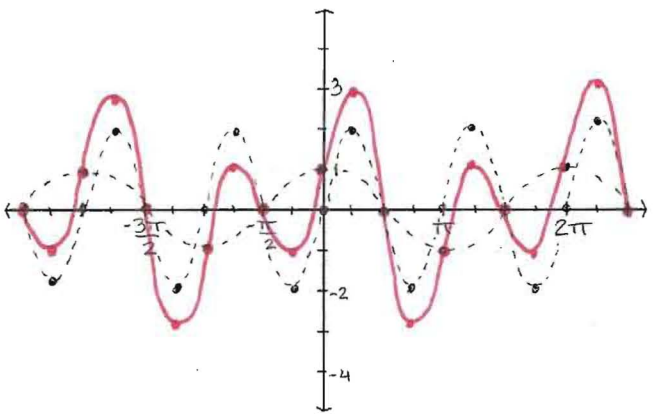
$y = \frac{1}{2}x - \cos x \quad = \frac{1}{2}x + (-\cos x)$

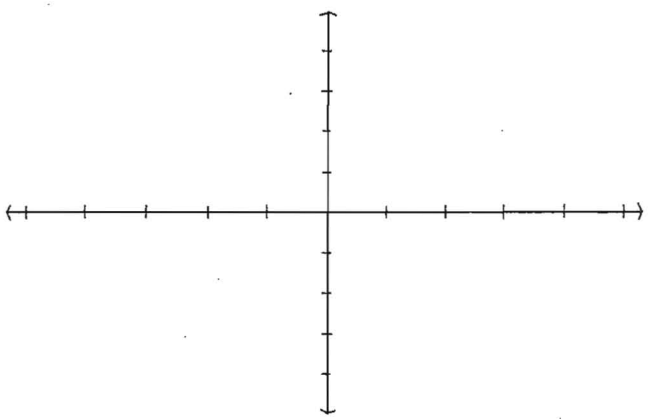


Extra examples:

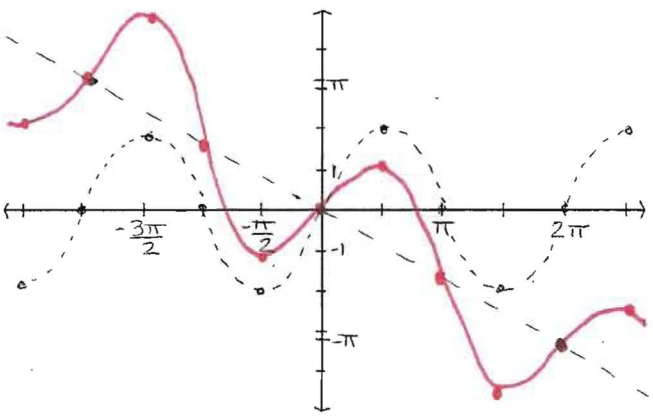
$y = 2\sin 2x + \cos x$

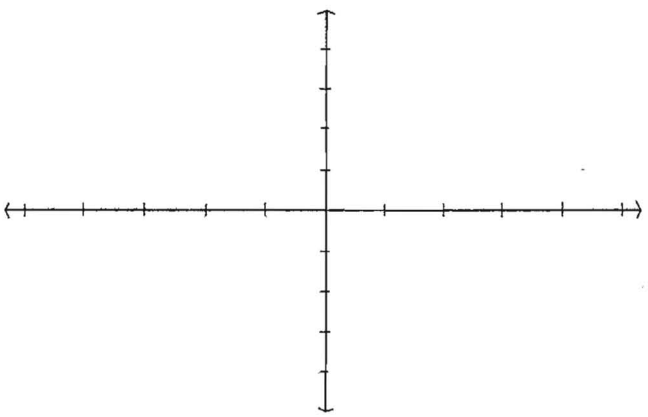
$2\sin 2x$ has amp 2 & per π
$\cos x$ has amp 1 & per 2π





$y = 2\sin x - \frac{1}{2}x$





$y = \cos 2x - 2\sin x$

$\cos 2x$ has amp 1 & per π
$2\sin x$ has amp 2 & per 2π