Guide to Writing and Presenting in Mathematics and the Arts

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Visual Mathematics
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Table of Contents

Topic: Choices, Choices, Choices

Research possibilities:

1. Architectural arches and domes
2. Bezier curves, splines, and computer graphics
3. Circle & sphere packing (Apollonian gasket)
4. Space-filling curves (Jordan curve theorem; Hilbert curves)
5. Classification of surfaces by genus and orientability (Conway's zip proof)
6. Hyperbolic geometry
7. The fourth dimension, Cubism, and Futurism
8. Tessellations (periodic & hyperbolic - Escher, aperiodic - Penrose)
9. Fibonacci and the Golden Ratio
10. Star Polygons and Islamic geometric patterns
11. Algorithmic art (Sol LeWitt)
12. Knots, links and Seifert surfaces
13. Origami crease pattern creation
14. Origami tessellations
15. Perspective drawing (1-point, 2-point, 3-point, 4-point, 6-point; Dick Termes)
16. Spherical symmetry (Temari fiber art)

Assignment:

Using Google or your favorite search engine, look up ALL of the suggested possible topics. In addition, browse through the journals, magazines, and other internet sources linked to from http://www.asms.net/library-research.php. Also, check out http://en.wikipedia.org/wiki/Mathematics_and_art.

Once you have at least some idea of the possibilities, choose your top three research topics, either from the list or your own topic(s) based on ideas you came across while browsing the web (please choose at least one topic from the list).

For each of these three topics, include links/references to internet sources that you found relating to those topics (at least two, but list as many as you found). These will not end up being actual sources for your final paper/presentation, but they are a good place to start. Typically, the more internet sources you can find for a particular topic, the more scholarly sources you will be able to find as well. Keep this in mind when choosing your topics, but above all, choose topics that interest you!

What to submit: A typed list of three topics with at least two internet sources for each.
How to submit it: Email your completed assignment to sgbrewer@dragons.asms.net.

Please make all links to internet sources active links so that I can easily click on them to scope out the source.

Name your file lastnamefirstname-researchtopics.doc (or .docx or .pdf).

Example:

Topic 1: Knitting hyperbolic surfaces
Sources:
- http://www.toroidalsnark.net/mkmisc.html#hp
- http://www.amazon.com/Crocheting-Adventures-Hyperbolic-Planes-Taimina/dp/1568814526
- http://theiff.org/oexhibits/oe1e.html

Topic 2: Mathematics and painting
Sources:
- http://www2.imperial.ac.uk/~hjjens/Math_Paint.pdf
- http://www.gap-system.org/~history/HistTopics/Art.html
- http://discovermagazine.com/2001/nov/featpollock

Topic 3: Scientific & engineering applications of origami
Sources:
- http://www.langorigami.com/science/science.php4
- http://www2.eng.cam.ac.uk/~sdg/preprint/5OSME.pdf
- http://web-japan.org/trends/07_sci-tech/sci080303.html
- http://math.serenevy.net/?page=Origami-ApplicationLinks
- http://esciencenews.com/articles/2009/02/27/mit.uses.nano.origami.build.tiny.electronic.devices

Outline: What's the point?

Assignment: Submit a 1-3 page outline of your research project. Include sources to back-up your points. If a particular sub-topic does not have any scholarly sources to back it up, you need to find some or not include that sub-topic. If a particular sub-topic has quite a few more scholarly sources than you expected (and that subtopic interests you), you may want to refine your research focus to those particular aspects of it that have more research behind them. The more details and subtopics you include, the better. The point of this assignment is to get a realistic view of the project you are undertaking and to help you organize your information as you collect it. Tips: Form your rough outline structure by referring to a general reference that may or may not be scholarly, e.g. the wikipedia article on the subject. Then refine your outline based on the sources you have found so far.

Minimum source requirements: 8 sources, at least one of which is a textbook and at least 3 of which are scholarly journal articles; no more than two non-scholarly internet sources; include at least three images to support your research, with detailed source information for images including URL if internet-sourced (image sources do not count toward "8 source" requirement)

How to submit it: Email your completed assignment as an attached file to sgbrewer@dragons.asms.net. Name your file lastnamefirstname-researchoutline.doc (or .docx or .pdf).

Example: "Knitting/Crocheting Hyperbolic Surfaces" - **Research Outline** filename: brewersarah-researchoutline.pdf

I. Introduction
    A. Who is knitting these things?
        1. Daina Taimina, Cornell University professor
    B. Why are they knitting them?
        1. suggestion by William Thurston (theiff.org, but need to find more scholarly source)

II. What is a hyperbolic surface? (Delman 20-22)
    A. Curvature (Taimina, 9-13)
    B. Methods of visualizing hyperbolic surfaces (Taimina, 133-136)

III. How does sculpture/fiber art help us to better understand hyperbolic surfaces?
    A. "Geodesics – or straight lines – on the hyperbolic surface can be sewn onto the crochet texture for easy
        examination." (theiff.org)
    B. properties of triangles (theiff.org)

IV. Other mathematical concepts nicely illustrated with the use of fiber arts
    A. Tiling/quilting (Belcastro)
    B. Graph Theory/embroidery (Belcastro 137-148)

Sources:

Anderson, James. *Hyperbolic Geometry*. Springer, 2005.

Belcastro, Sarah-Marie. *Making Mathematics with Needlework: Ten Papers and Ten Projects*. AK

Peters, 2008.
Delman, Charles and Gregory Galperin. "A Tale of Three Circles," *Mathematics Magazine*, Vol. 76 No. 1 (Feb 2003), pp. 15-32.
"Hyperbolic Space Crochet Models," online exhibit, http://theiff.org/oexhibits/oe1e.html
Taimina, Daina. *Crocheting Adventures with Hyperbolic Planes*. AK Peters, 2009.
Taimina, Daina and David Henderson. "Crocheting the hyperbolic plane," *Mathematical Intelligencer*, Vol. 23, No. 2 (Spring 2001),
pp. 17-28. Available from:
http://www.math.cornell.edu/~dwh/papers/crochet/crochet.html
Wertheim, Margaret. *A Field Guide to Hyperbolic Space: An Exploration of the Intersection of Higher Geometry and Feminine*
Handicraft. The Institute for Figuring, 2007.
York, Michelle. "Professor lets her fingers do the talking," *The New York Times*, 13 March 2005. Available from
http://theiff.org/press/NYTcrochet.html.

Notes & Quotes: Turning Your Sources into Notes You Can Use

"Crocheting Hyperbolic Surfaces" **Notes & Quotes** 13 December 2012

Klarreich, Erica **. "CRAFTY GEOMETRY**," *Science News*. 12/23/2006, Vol. 170 Issue 26/27, p411-413.

"In a flat plane or a sphere, the circumference of a circle grows at most linearly as the radius increases. By contrast, in the hyperbolic plane, the circumference of a circle grows exponentially. As a result, the hyperbolic plane is somewhat like a carpet that, too big for its room, buckles and flares out more and more as it grows.

In 1901, mathematician David Hilbert proved that because of this buckling, it's impossible to build a smooth model of the hyperbolic plane. His result, however, left the door open for models that are not perfectly smooth.

In the 1970s, William Thurston, now also at Cornell, described a way to build an approximate physical model of the hyperbolic plane by taping together paper arcs into rings whose circumferences grow exponentially. However, these models take many hours to build and are so fragile that they generally need to be protected from much rough-and-tumble hands-on study.

Taimina realized that she could crochet a durable model of the hyperbolic plane using a simple rule: Increase the number of stitches in each row by a fixed factor, by adding a new stitch after, for instance, every two (or three or four or n) stitches. In 2001, Taimina and her Cornell colleague David Henderson proved that the crocheted objects indeed capture the geometry of the hyperbolic plane."

"Because the hyperbolic plane is so hard to visualize, Taimina's crocheted models are helping even seasoned mathematicians develop a better intuition for its properties. Taimina recalls that one mathematician, upon examining one of her hyperbolic planes, exclaimed, 'So that's what they look like!'"

Weschler, Lawrence. "The Hyperbolic Crochet Coral Reef." *Virginia Quarterly Review*; Summer2011, Vol. 87 Issue 3, p124-139.

"the early nineteenth century when a series of geometrists began to suspect the efficacy of Euclid's fiffh axiom—the one that asserts that on any given surface if you have a straight line and a point outside that line, there is one and only one line that can go through that outer point and not intersect the first line (the so-called parallel postulate). First Gauss and then Lobachevsky and Bolyai and presently Riemann and Hubert all began saying, effectively, "Oh, yeah, what about this line and this line and this line?" (abstractly, algebraically understood)—in fact one could envision a coherent mathematics in which infinite lines passed through that first point without intersecting that line (and it's a very fertile field of mathematics at that, one that eventually spawned everything

from the theory of relativity to much of the infrastructure of the Web). The only problem is, it's damnably difficult to envision, let alone rezdize, the precise sort of surface any such mathematics could actually be operating o. Mathematicians during the nineteenth and into the twentieth century famously drove themselves crazy—in some cases, literally so—trying to do so. That is until about twenty years ago when (long story, expertly conveyed on the Institute For Figuring's website at theiff. org) a Latvian émigré mathematician named Daina Taimina teaching at Cornell heard about the problem from her geometer husband and effectively said, "What's the big deal? You can crochet such a surface." And, drawing on the women's handicraft traditions of her childhood, she proceeded to do just that. (Such is the cost of having had so few women mathematicians until relatively recently.)"

*article contains great images!

Henderson, David W. and Taimina, Daina. "Crocheting the Hyperbolic Plane." *Mathematical Intelligencer*, Spring2001, Vol. 23 Issue 2, p17.

"it is impossible to embed the hyperbolic plane isometrically (an isometry is a function that preserves all distances) as a complete subset of Euclidean 3-space"

"All of the references are implicitly assuming surfaces embedded with some conditions of differentiability"

Samuels, David. 2006. "KNIT THEORY." *Discover* 27, no. 3: 40-43. *Academic Search Complete*, EBSCO*host* (accessed December 13, 2012).

"Hyperbolic geometry describes a world that is curving away from itself at every point, making it the precise opposite of a sphere, whatever that might look like."

"Hyperbolic geometry describes a world that is curving away from itself at every point, making it the precise opposite of a sphere, whatever that might look like."

Shilling, Jane. "Hyperbolic crochet." *Times, The (United Kingdom)*, 06/13/2008 *Newspaper Source*, EBSCO*host* (accessed December 13, 2012).

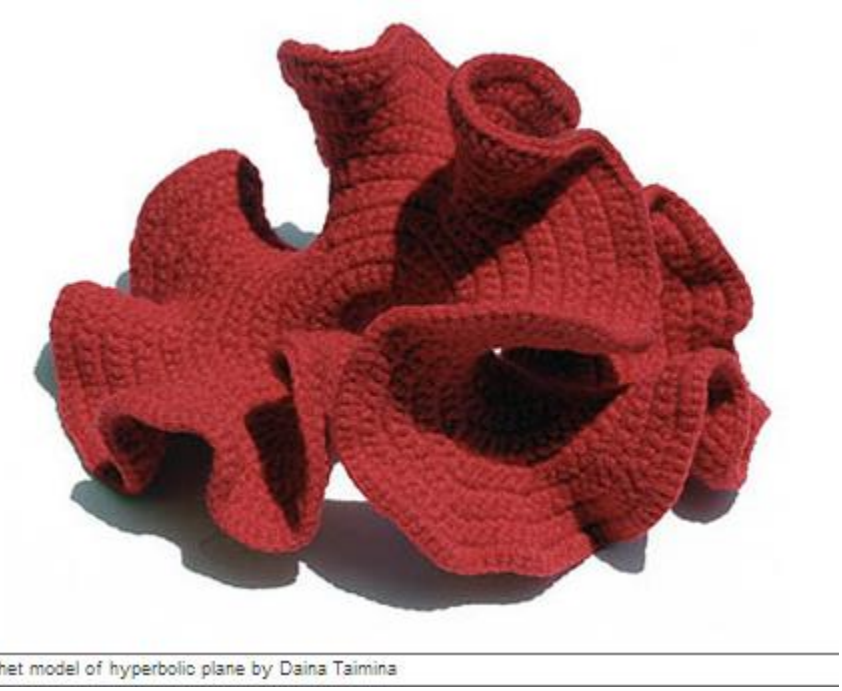
"twin sisters, Margaret and Christine Wertheim, who grew up in Queensland and have crocheted a model of the Great Barrier Reef…. The Wertheim sisters are academics - Margaret a physicist and mathematician, Christine a lecturer in feminism and popular culture. In 2003 they founded the Institute for Figuring, the aim of which is to explore the "poetic and aesthetic" aspects of science."

Alex, Bellos. n.d. "It's crochet, Jim, but not as we know it." *Times, The (United Kingdom)*, n.d. *Newspaper Source*, EBSCO*host* (accessed December 13, 2012).

"It is so conceptually challenging that for a century mathematicians were unable to visualise what this type of space might actually look like. In fact, there is no formula that accurately describes hyperbolic space, so computers cannot model it either."

"Hyperbolic geometry maximises area with a minimal volume. Taimina's giant coral has a surface area of about 3.2 square metres (35 sq ft). "This is about twice as much as me," she says. When an organism needs a large surface area, for example a filter-feeding organism like coral, it grows in a hyperbolic way."

"Hyperbolic Space online exhibit." The Institute for Figuring. http://theiff.org/oexhibits/oe1e.html



Crochet model of hyperbolic plane by Daina Taimina

"The beauty of Taimina's method is that many of the intrinsic properties of hyperbolic space now become visible to the eye and can be directly experienced by playing with the models. Geodesics – or straight lines – on the hyperbolic surface can be sewn onto the crochet texture for easy examination. Through the yellow lines in the model below look curved, folding along them demonstrably produces a clean straight line."

"Likewise one can see immediately how the parallel postulate is violated. In the model below there are three straight lines that pass through a point external to a given line (the one at the bottom). All three of the upper lines never intersect the original line. With this physical model in your hands you can fold along each line and verify the untruth of Euclid's fifth axiom yourself. The power of Taimina's models resides in bringing abstract mathematics into the realm of tactile experience."

"Here is the rate of increase is one stitch in every three."

"Here we are increasing one stitch in every two."

"And here, we are increasing in every stitch."

“As the rate of increase of the stitches increases, the construction becomes increasingly crenellated. In effect the space around any point expands ever more exponentially. Whereas all spheres have the same form - varying only in size - hyperbolic surfaces may differ dramatically from an external point of view. Mathematically speaking, we characterize this quality by the radius of the hyperbolic plane: the more crenellated the surface appears, the smaller its radius; the flatter the surface, the greater its radius. A regular Euclidean plane can be understood as a hyperbolic plane of infinite radius. The radius of any given hyperbolic plane is the radius of a circle that would sit flatly on an a tabletop.

In the series of pseudospheres above, the radii get progressively smaller, leading to ever more crenellated models. By using a high rate of expansion - by increasing in every stitch, or even increasing two or three times in every stitch - we can make a hyperbolic surface that starts off from just a few stitches and expands out to an arbitrarily long perimeter. Dr Taimina’s personal record is a plane that began with two dozen stitches and has a perimeter length of 369 inches. The width of this model is just 4 inches and the total weight of wool is nearly a pound. Dr Taimina’s models are featured in the Smithsonian's collection of American Mathematical Models.”

Rough Draft: Getting Something Down on Paper

Sarah Brewer "Crocheting Hyperbolic Planes" 14 November 2013

All of Geometry is based on five postulates set forth by Euclid in (date?), the fifth of which is the famous "Parallel Postulate." The parallel postulate basically states that through a given point not on a given line, there is exactly one line that can be drawn through that point which will never intersect the given line. While this holds true in flat Euclidean geometry, there are instances when this is not true. David Hilbert and others discovered geometries in which there are many lines that can be drawn through a given point not on a given line which will not intersect that line. That geometry with which we are concerned is called Hyperbolic Geometry.

In a hyperbolic plane, space curls away from itself (citation?). In this way, we can think of a hyperbolic plane as being the opposite of a sphere (citation?). If this seems hard to picture in your mind, that's because it is. For centuries, mathematicians struggled with the problem of trying to visualize these strange surfaces, and it wasn't until Thurston found a way to construct a paper model in the 1970s that mathematicians were finally able to study certain properties of hyperbolic planes that could not be seen algebraically. The construction of the hyperbolic plane is so complicated, in fact, that no computer program has been able to model them accurately (citation?).

When Taimina was given the job of teaching a Hyperbolic Geometry class at (?) University in (date?), she too struggled with the idea of how to describe to her students just what exactly a hyperbolic plane looks like (citation). Dissatisfied with Thurston's paper models, which are quite delicate and not meant for getting one's hands dirty with math (LAME – reword!), she turned to her treasured pastime of crochet. By crocheting a circle, and increasing the number of stitches in a

systematic way as you go out, the edges of the circle begin to bend and fold. The radius of the circle is increasing exponentially (citation).

The sturdy crochet models that Taimina was able to produce are ideal for studying hyperbolic geometry especially because of their ability to bend and fold. One can stitch lines onto the surface and see how the hyperbolic plane really does defy Euclid's fifth postulate (citation).

Figure 1. Crochet model contradicting Euclid's Parallel Postulate.

http://theiff.org/oexhibits/oe1e.html

Final Draft: Something to be Proud of

Short Research Paper Guidelines (15 November 2013)

- "normal" 1-inch margins on all sides
- **For one-page papers, 350-500 words** of 1.5-2- spaced 10-12-pt serif font (e.g. Cambria or Times New Roman) ; **for two-page papers, 700-1000 words**
- Name, title, and date must appear above essay (for one-page papers) or on a cover page
- For one-page papers, try to fit all of the text of the essay on the first page by changing font, font size, and/or spacing; images may be included after the text on a second page; for longer papers, include images within text.
- Be sure to include **citations** for all information that is not common knowledge in the form of either endnotes-bibliography or author-date parenthetical references. See style guide for examples.
- Works cited/bibliography should contain all sources, properly cited in Chicago/Turabian style, as well as small (but legible) versions of any images referred to in the text, also properly cited. The Works Cited includes only sources referred to specifically in the text of your paper (and image sources). The Bibliography includes all sources available on your subject that you read while conducting your research, even if you did not end up referring to these directly in your paper.
- Write entirely in the **active voice**. For examples of active v. passive voice, please see the Purdue Online Writing Lab.
- Make a point in your paper, and support it with evidence. This is not a book report. Structure and style count.

- Include **images** if they help you make your point; these must also be cited. Annalisa Crannell has a great guide to formatting figures within a paper.
- **Sources must be scholarly**, and should include mostly peer-reviewed journals and textbooks. Primary source artist statements and conference proceedings are also acceptable. Professor-generated summaries are okay if they are properly cited, but only if you can't access the sources that they used. Quality popular sources (Wikipedia, New Scientist and other respected magazines and/or blogs) may be included in your bibliography as references to give you an understanding of the topic, but should not be relied upon as source material.
- Papers will be submitted via **turnitin.com**
- Don't write your assignment the night before it is due. Bring it to the writing lab at *least* once. Submit it to turnitin.com at *least* once before your final submission.

Paper Title

Your Name

Course Name
Instructor's Name
Date Submitted

This is an attention-grabbing sentence that is relevant to my paper topic. The rest of this paragraph serves to introduce the topic and give the reader a sense of what they can expect to learn more about in the rest of the paper. It may contain some facts, but most of them will probably be more of the general knowledge type than specific facts that require citations.

The next paragraph should serve to define your topic in mathematical terms, and might be boring to the lay reader. (Author1, p.1) It is important to be accurate in your mathematical descriptions, and include both the technical explanation, followed by an explanation in your own words. Images often help to explain complicated mathematical ideas. (see Figure 1) Images should be placed in the paper at the end of a paragraph, and should only be included if they are specifically referred to in the paper and help to explain the topic. (Author 2, p.1)

Figure 1. Description of image, Internet Source 1.

This paragraph is about my first example. Note that any information that is not considered general knowledge should include a citation for the source where I read it. ("What", p.3) "Any direct quotes should include a citation, even if the following sentence is from the same source." ("Who" p.5) If you have two different articles by the same author, it is often convenient to cite them in the text of your paper by a shortened version of the article title, rather than the author. ("Who" p.6)

This paragraph is about my second example. It is a really good example, but there has been limited research done on it, so all of the information in this paragraph is from the same source, and I am not including any direct quotes from that source. Therefore, the only citation will occur at the end of the paragraph. (Author 4, pp.9-10)

This paragraph is about my third example. Three examples, if explained well, is a good number for most topics. (Author 1, p.6) Trying to cover too much material might result in less in-depth research for the examples that you have. Focus on understanding a small amount of material rather than trying to understand everything. (Author 5, p.9)

This is my conclusion paragraph. It will probably contain some generalization of things that I learned in doing my research, and may also mention some examples or topics related to my paper that I didn't get the chance to write about, but that the reader may want to research on their own if they find my topic interesting and would like to learn more.

Works Cited

Author 1, "Title of Article," *Name of Journal*, Volume #, Issue #, page numbers used.

Author 2, *Book Title*, Publisher, year published, page numbers used.

Author 3, "What," *Name of Journal*, Volume #, Issue #, page numbers used.

Author 3, "Who," *Name of Journal*, Volume #, Issue #, page numbers used.

Author 4, "Title of Article," *Name of Journal*, Volume #, Issue #, page numbers used, available from JSTOR, accessed on date.

Author 5, "Title of Article," *Name of Journal*, Volume #, Issue #, page numbers used.

"Internet Source 1 not credited to any author because there was no author listed," URL, accessed on date.

Bibliography

Author 1, "Title of Article," *Name of Journal*, Volume #, Issue #.

Author 2, *Book Title*, Publisher, year published.

Author 3, "What," *Name of Journal*, Volume #, Issue #.

Author 3, "Who," *Name of Journal*, Volume #, Issue #.

Author 4, "Title of Article," *Name of Journal*, Volume #, Issue #, page numbers used, available from JSTOR, accessed on date.

Author 5, "Title of Article," *Name of Journal*, Volume #, Issue #.

"Internet Source 1 not credited to any author because there was no author listed," URL, accessed on date.

"Wikipedia article that I didn't reference in my paper because it's not scholarly but that I read to help me understand my topic," *Wikipedia*, URL, accessed on date.